جامعة دمشق

كلية العلوم - قسم الرياضيات

(إيجاد التشاكلات الفئوية المثبتة للعناصر للفئة G-R-P-H)

(FINDING FIXOBS FOR G-R-P-H)

دراسة أعدت لنيل درجة الماجستير في الرياضيات من جامعة دمشق

دمشق . الجمهورية العربية السورية

إعداد الطالبة : نور حسين

اشراف

أ. **م .د فرحان اسماعيل**

العام الدراسي 2009- 2010

Contents الفهرس

الفصل الأول

مفاهيم أساسية

المقدمة :

نقوم في هذا الفصل بعرض عدد من التعاريف و الملاحظات الهامة في كل من نظرية البيان (Graph Theory) و نظرية الفئات (Category Theory)

بما في ذلك البيانات (Graphs) و البيانات الموجهة (DIGRAPH) و الفئات (Category)

و التشاكلات الفئوية و خاصة التشاكلات الفئوية المثبتة للعناصر (Fixob) مع عرض عدد من الأمثلة عن كل منها .

(1-1) البيانات (Graphs) :

(1-1-1) تعريف:

البيان (G (Graph هو مجموعة منتهية Vمن العقد و جماعة E من أزواج العقد تدعى أضلاع (edges).

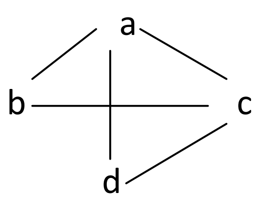
(1-1-2) ملاحظة :

تمثل العقد بنقط وتمثل الأضلاع (edges) بخطوط (ليست بالضرورة مستقيمة) تصل بين أزواج العقد .

(1-1-3) مثال :

البيان الممثل بالشكل جانبا

مؤلف من أربع عقد و من خمسة أضلاع



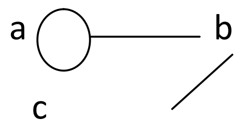
(1-1-4) تعريف:

العروة (LOOP) هي ضلع يصل العقدة x مع نفسها .

(1-1-5) مثال :

البيان الممثل في الشكل

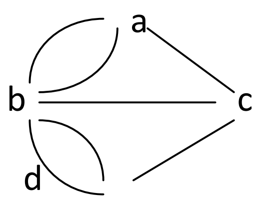
يحوي عروة (LOOP) عند العقدة a



(6-1-1) تعريف:

الضلع المضاعف (multiple edge) هي ضلعين أو أكثر تصل بين عقدتين .

(7-1-1) مثال:



البيان الممثل بالشكل

يحوي زوجين من الأضلاع المضاعفة (multiple edge) .

(8-1-1) تعريف:

نقول عن بيان (graph) أنه بسيط (simple graph) إذا كان لا يوجد فيه أي عروة أو أي ضلع مضاعف .

(9-1-1) تعريف:

درجة العقدة هي عدد الأضلاع (edges) المتعلقة بهذه العقدة في البيان البسيط (بدون LOOP)

(10-1-1) ملاحظة :

العقدة التي درجتها 1 تدعى end أوpendant .

(11-1-1)نظرية :

مجموع درجات العقد في البيان هو ضعفي عدد الأضلاع (edges) [1]

(12-1-1) تمهيدية :

في أي بيان (graph) مجموع درجات العقد هو عدد زوجي . [1]

(13-1-1) تعريف :

نقول عن graph (H) أنه بيان جزئي (sub graph) من البيان (G) (graph) إذا كانت :

– مجموعة عقد H هي مجموعة جزئية من مجموعة عقد G .

- مجموعة الأضلاع (edges) للبيان الجزئي H هي مجموعة جزئية من مجموعة الأضلاع (edges) للبيان G .

(14-1-1) تعريف :

التشاكل البياني $D:G \longrightarrow G^*$ حيث G و * G بيانين (graphs)

هو زوج من الدوال : $D_A: A \longrightarrow A^*$

$D_O: O \longrightarrow O^*$

حيث لأجل كل ضلع f يكون :

$D_O \sigma_1 f = \sigma_1 D_A f$ $\quad$ $D_O \sigma_0 f = \sigma_0 D_A f$

حيث O مجموعة عناصر البيان G وA مجموعة أضلاع البيان G .

و * O مجموعة عناصر البيان * G و * A مجموعة أضلاع البيان * G .

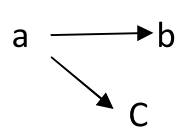
(2-1) البيانات الموجهة (DIGRAPH).

(1-2-1) تعريف:

البيان الموجه(digraph) هو عبارة عن بيان (graph) يعبر فيه عن كل ضلع(edge) بسهم (ar c).

(2-2-1) مثال :

البيان الموضح جانبا هو عبارة عن بيان موجه (digraph)



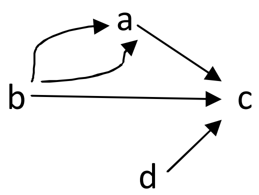
ملاحظة :

في البيان الموجه العروة (LOOP) هي عبارة عن سهم يصل العقدة مع نفسها .

(4-2-1) تعريف :

نقول عن سهمين أو أكثر يصلان نفس العقدة و بنفس الاتجاه أنها أسهم مضاعفة (multiple arcs).

(1-2-5) مثال:



البيان المعطى بالشكل المقابل

يحوي زوج من الأسهم المضاعفة (multiple arc).

(1-2-6) تعريف:

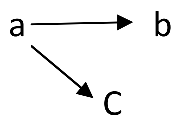
البيان الموجه الجزئي (subdigraph) من digraph (D) هو digraph جميع عقده هي عقد من (D) و جميع أسهمه هي أسهم من(D) .

(1-2-7) تعريف :

indegree : عدد الأسهم الموجهة باتجاه العقدة ،

outdegree : عدد الأسهم الخارجة من العقدة .

(1-2-8) مثال :



في البيان الموجه الموضح جانبا

indegree للعقدة b هو 1

outdegree للعقدة a هو 2

(1-2-9) مبرهنة : [1]

في أي بيان موجه يكون مجموع indegree لجميع العقد مساوياً لمجموع outdegree لجميع العقد ويساوي عدد الأسهم في البيان .

(3-1) الفئات(Categories)

(1-3-1) تعريف:

الفئة هي عبارة عن مجموعة O من الأشياء و مجموعةA من الأشعة و تابعين :

$\text{Dom} : A \longrightarrow O$ $\qquad$ $\text{Cod} : A \longrightarrow O$

$f \longrightarrow \text{dom } f$ $\qquad$ $f \longrightarrow \text{cod } f$

مع العمليتن :

$\text{Id} : O \longrightarrow A$ $\qquad$ $AX_O A \longrightarrow A$

$c \longrightarrow id_c$ $\qquad$ $(g,f) \longrightarrow gof$

و هاتان العمليتان تحققان الخاصتين : التجميعية و التركيب مع شعاع الوحدة .

حيث $AX_O A$ الجداء فوق O هو مجموعة أزواج الأشعة القابلة للتركيب :

$$AX_O A = \{ (g, f) / g, f \in A \ , \ \text{dom } g = \text{cod } f \}$$

(2-3-1)أمثلةعلى الفئات :

1)الفئة Set: فئة المجموعات :

- أشياؤها كل المجموعات
- أشعتها كل التطبيقات .

التركيب هو تركيب التطبيقات

2) الفئة Grp: فئة الزمر حيث :

- أشياؤها كل الزمر
- أشعتها كل التشاكلات الزمرية

3) الفئة Grph فئة البيانات(graphs) حيث :

- أشياؤها كل البيانات (graphs)
- أشعتها كل التشاكلات البيانية

4)الفئات $Grph_{<n}$: حيث أن $n \geq 0$

- أشياؤها البيانات المولدة بالمجموعات $n=0,1,2,\ldots\ldots.n-1$
- أشعتها التشاكلات البيانية بين البيانات السابقة .

(3-3-1) تعريف :

التشاكل الفئوي (Functor) هو عبارة عن تابع بين فئتين

$$T: A \longrightarrow B$$



ينقل كل عنصر من A إلى عنصر من B و كل شعاع f من A إلى شعاع Tf من B.

و بحيث يحقق : $T(1_a) = 1_{Ta}$

$$T(fog) = Tf \circ Tg$$

ملاحظة:

يدعى التشاكل $F: C \longrightarrow C$ واحدياً ويرمز له II_{C} إلى نقل كل عنصر إلى نفسه وكل شعاع إلى نفسه

(4-3-1) تعريف:

لتكن A فئة ما ، نقول عن التشاكل الفئوي $F: A \longrightarrow A$

إنه تشاكل مثبت للعناصر (Fixob) إذا كانت صورة أي عنصر a من A هو نفسه
أي $F(a) = a$ وذلك من أجل كل a من A

(5-3-1) ملاحظة :

نقول إن التشاكل الفئوي المثبت للعناصر F متماثل مع التشاكل الفئوي الواحدي للفئة A

إذا كان $F(f) = f$ و ذلك من أجل كل شعاع f من A

الفصل الثاني

البيانات المولدة بالمجموعات المنتهية 0,1,2,3,4,5,6

نقوم في هذا الفصل بتوليد البيانات من المجموعات المنتهية 0,1,2,3,4,5,6

و ذلك من خلال خوارزمية إضافة ضلع جديد إلى البيان و خوارزمية أخرى لتوليد البيانات ومن ثم تحديد و حذف البيانات المتكافئة للتوصل إلى تجميع البيانات في صفوف تكافؤ و استنتاج عدد هذه الصفوف ونشير هنا إلى أننا سنستخدم كلمة البيان ليدل على البيان الموجه البسيط (لا يوجد تكرار للأضلاع) .

(1-2)اصطلاحات :

- نرمز $G_{n,m}$ للبيان المؤلف من n رأس وm ضلع .
- نرمز $G^{k}_{n,m}$ لصف التكافؤ k للبيان المؤلف من n رأس وm ضلع .
- indegree : عدد الأسهم الموجهة باتجاه العقدة ،
- outdegree : عدد الأسهم الخارجة من العقدة .
- البيان المولد بالمجموعة n أي بيان نحصل عليه من n رأس (أو العقدة)

(2-2) خوارزمية إضافة ضلع جديد :

1- أعبر عن البيان (Graph) بإيجاد متتالية indegree و outdegree لكل عقدة .

2- عند إضافة ضلع جديد للبيان (Graph) مثل ad فهو إما

d ⟶ a   a ⟶ d

3- في حالة إضافة الضلع a ⟶ d

- أضيف العدد (1) عند outdegre للعقدة a في متتالية indegree و outdegree
- و أضيف العدد (1)عند indegre للعقدة d في متتالية indegree و outdegree

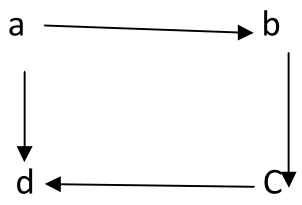
4- في حالة إضافة الضلع d ⟶ a

- أضيف العدد (1) عند indegre للعقدة a في متتالية indegree و outdegree
- وأضيف العدد (1) عند outdegre للعقدة d في متتالية indegree و outdegree

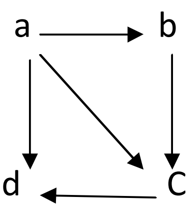
(1-2-2) مثال :

ليكن لدينا البيان (Graph) $G_{4,4}$

	in	out
a	0	2
b	1	1
c	1	1
d	2	0

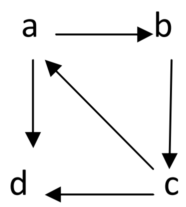


فعند إضافة الضلع الجديد ac نحصل على :



	in	out
a	0	3
b	1	1
c	2	1
d	2	0

أو



	in	out
a	1	2
b	1	1
c	1	2
d	2	0

(3-2) خوارزمية ايجاد البيانات :

1- نوجد $G_{n,0}$ - $G_{n,1}$ باعتبار أنها حالات بسيطة .

2- نعبر عن البيان (Graph) بمتتالية من indegree و outdegree لكل عقدة .

3- نحدد جميع الأضلاع الممكن اضافتها لحالة $G_{n,1}$

4- نأخذ أحد حالات $G_{n,1}$ و نضيف إليها ضلعا من هذه الأضلاع الممكنة و نعبر عنها بمتتلية indegree و outdegree .

5- نكرر العملية لإضافة جميع الأضلاع الممكنة.

6- نكرر العملية لجميع حالات $G_{n,1}$.

7- نحذف الحالات المتماثلة و ذلك اعتمادا على متتالية indegree و outdegree .

و هكذا إلى أن نصل إلى بيانات حالة $G_{n,2}$.

8- نعيد خطوات العمل رقم (3-4 -5-6) لإيجاد بقية البيانات $G_{n,3}$ – $G_{n,4}$ -------

(1-2-2) مثال :

- $G_{3,0}$

a b

c

	In	out
a	0	o
b	0	0
c	0	0

- $G_{3,1}$

a ⟶ b

c

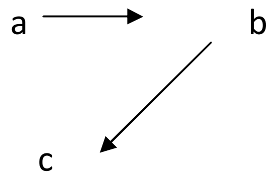
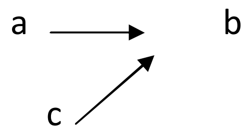
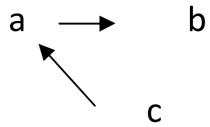
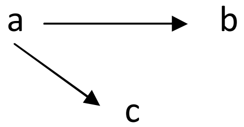
	In	out
a	0	1
b	1	0
c	0	0

- لإيجاد $G_{3,2}$ فإن جميع الأضلاع الممكنة وهي

(a ⟶ c) (c ⟶ a) (b ⟶ c) (c ⟶ b)

- نضيف كل ضلع منها على حدى للبيان $G_{3,2}$ فنحصل على :

-1	-2
In out	In out

 a 0 1 b 1 1 c 1 0	 a 0 1 b 2 0 c 0 1
-4 In out  'a 1 1 b 1 0 c 0 1	-3 In out  0 2 1 0 1 0

فنجد أن الحالة رقم 2 و رقم 4 متكافئتين فنحذف أحدهما و بالتالي حالة $G_{3,2}$ مكونة من ثلاثة بيانات

- نعيد العملية لإيجاد$G_{3,3}$.

بتطبيق هذه الخوارزمية على المجموعات n= 0,1,2,3,4,5,6 توصلنا إلى النتائج التالية :

(4-2) البيانات المولدة بالمجموعات 0,1,2,3,4,5,6 :

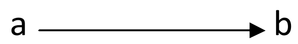
(2-4-1) البيانات المولدة بالمجموعة 0 (G_0) :

أي عدد الرؤوس 0 و عدد الأشعة 0 عندها لا يوجد أي بيان .

(2-4-2) البيانات المولدة بالمجموعة 1 (G_1) :

- $G_{1,0}$ حيث عدد الرؤوس 1 و عدد الأشعة 0 سنرمز له

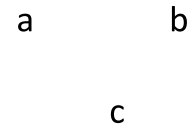
(2-4-3) البيانات المولدة بالمجموعة 2 (G_2) :

$G^1_{2,0}$	a b
$G^1_{2,1}$	

(4-4-2)البيانات المولدة بالمجموعة 3 (G_3) :

1) $G_{3,0}$

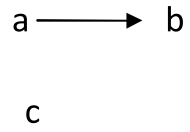
$G^1_{3,0}$



	In	out
a	0	o
b	0	0
c	o	0

2) $G_{3,1}$

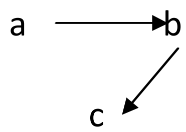
$G^1_{3,1}$



	In	out
a	0	1
b	1	0
c	0	0

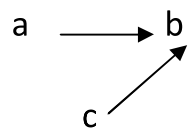
3) $G_{3,2}$

$G^1_{3,2}$



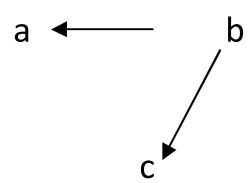
	In	out
a	0	1
b	1	1
c	1	0

$G^2_{3,2}$



	In	out
a	0	1
b	2	0
c	0	1

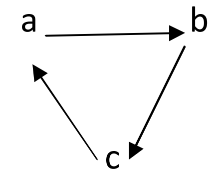
$G^3_{3,2}$



	In	out
a	1	0
b	0	2
c	1	0

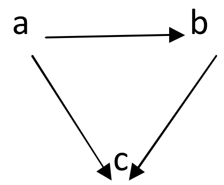
$G_{3,3}$ (4

$G^1_{3,3}$



	In	out
a	1	1
b	1	1
c	1	1

$G^2_{3,3}$



	In	out
a	0	2
b	1	1
c	2	0

(5-4-2)البيانات المولدة بالمجموعة 4 (G_4) :

$G_{4,0}$ (1

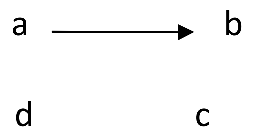
$G^1_{4,0}$



	In	out
a	0	0
b	0	0
c	0	0
d	0	0

$G_{4,1}$ (2

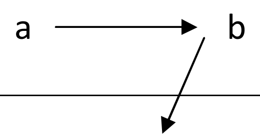
$G^1_{4,1}$



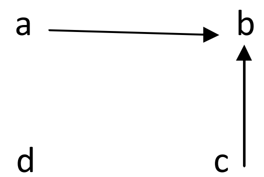
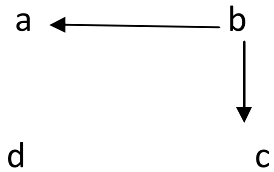
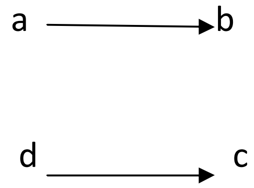
	In	out
a	0	1
b	1	0
c	0	0
d	0	0

$G_{4,2}$ (3

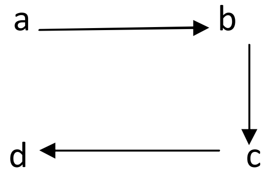
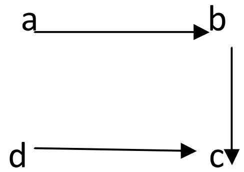
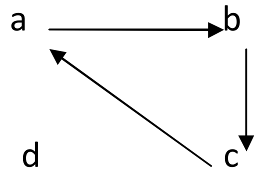
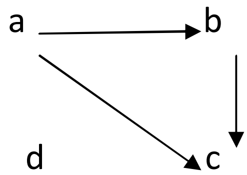
$G^1_{4,2}$



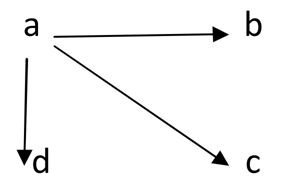
	In	out
a	0	1
b	1	1

			In	out
	d c	c d	1 0	0 0
$G^2_{4,2}$		a b c d	0 2 0 0	1 0 1 0
$G^3_{4,2}$		a b c d	1 0 1 0	0 2 0 0
$G^4_{4,2}$		a b c d	0 1 1 0	1 0 0 1

$G_{4,3}$ (4

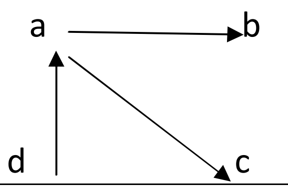
			In	out
$G^1_{4,3}$		a b c d	0 1 1 1	1 1 1 0
$G^2_{4,3}$		a b c d	0 1 2 0	1 1 0 0
$G^3_{4,3}$		a b c d	1 1 1 0	1 1 1 0
$G^4_{4,3}$		a b c d	0 1 2 0	2 1 0 0

$G^{5}_{4,3}$



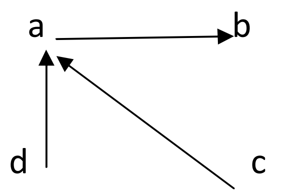
	In	out
a	0	3
b	1	0
c	1	0
d	1	0

$G^{6}_{4,3}$



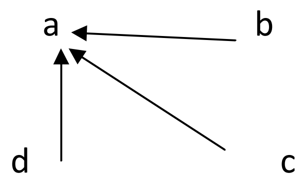
	In	out
a	1	2
b	1	0
c	1	0
d	0	1

$G^{7}_{4,3}$



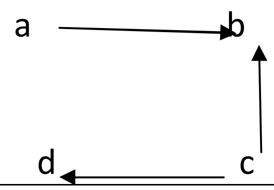
	In	out
a	2	1
b	1	0
c	0	1
d	0	1

$G^{8}_{4,3}$



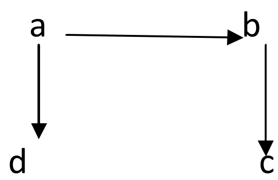
	In	out
a	3	0
b	0	1
c	0	1
d	0	1

$G^{9}_{4,3}$



	In	out
a	0	1
b	2	0
c	0	2
d	1	0

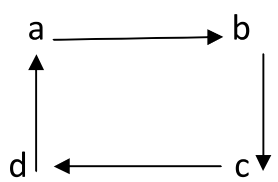
$G^{10}_{4,3}$



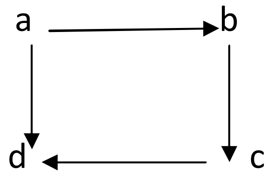
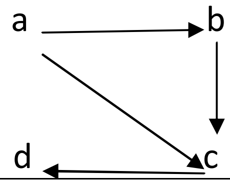
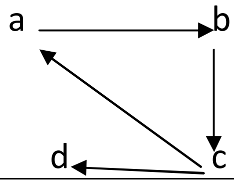
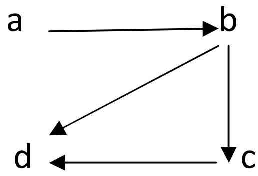
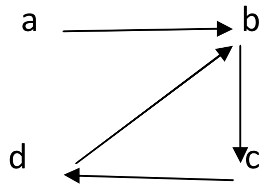
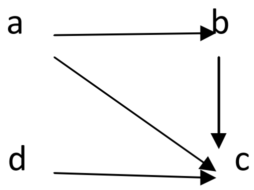
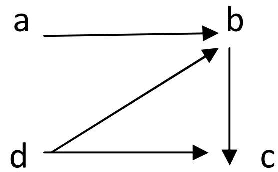
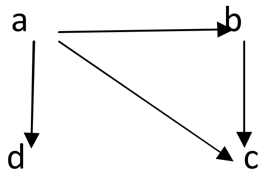
	In	out
a	0	2
b	1	1
c	1	0
d	1	0

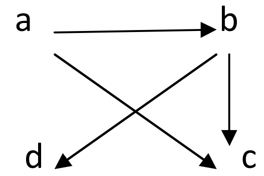
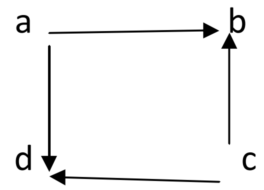
$G_{4,4}$ (5

$G^{1}_{4,4}$

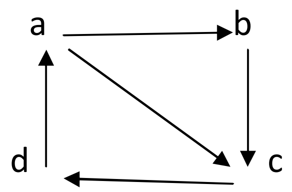
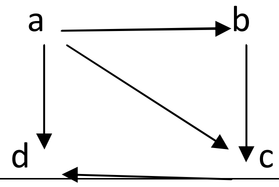
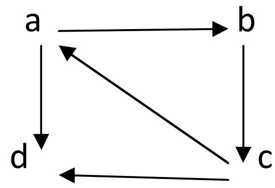
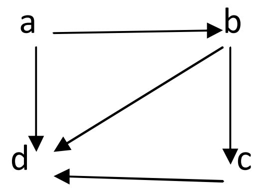
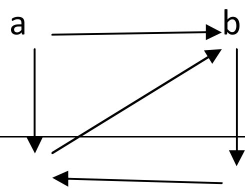


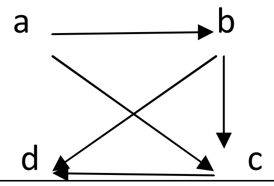
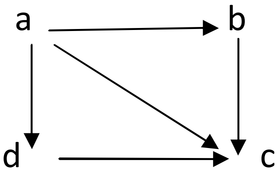
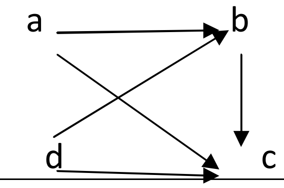
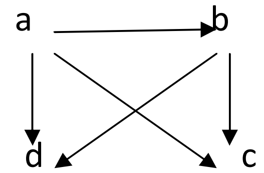
	In	out
a	1	1
b	1	1
c	1	1
d	1	1

Graph	Diagram		In	out
$G^2_{4,4}$			In	out
		a	0	2
		b	1	1
		c	1	1
		d	2	0
$G^3_{4,4}$			In	out
		a	0	2
		b	1	1
		c	2	1
		d	1	0
$G^4_{4,4}$			In	out
		a	1	1
		b	1	1
		c	1	2
		d	1	0
$G^5_{4,4}$			In	out
		a	0	1
		b	1	2
		c	1	1
		d	2	0
$G^6_{4,4}$			In	out
		a	0	1
		b	2	1
		c	1	1
		d	1	1
$G^7_{4,4}$			In	out
		a	0	2
		b	1	1
		c	3	0
		d	0	1
$G^8_{4,4}$			In	out
		a	0	1
		b	2	1
		c	2	0
		d	0	2
$G^9_{4,4}$			n	out
		a	0	3
		b	1	1
		c	2	0
		d	1	0

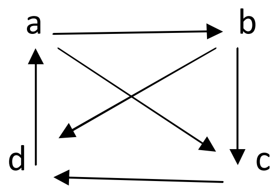
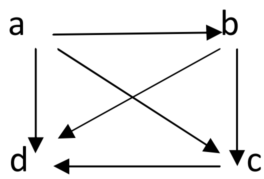
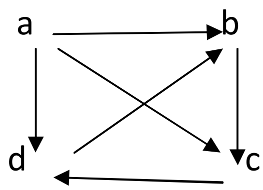
Graph	Drawing	Vertex	In	out
$G^{10}_{4,4}$		a	0	2
		b	1	2
		c	2	0
		d	1	0
$G^{11}_{4,4}$		a	0	2
		b	2	0
		c	0	2
		d	2	0

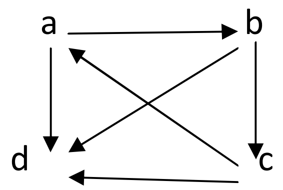
$G_{4,5}$ (6

Graph	Drawing	Vertex	In	out
$G^{1}_{4,5}$		a	1	2
		b	1	1
		c	2	1
		d	1	1
$G^{2}_{4,5}$		a	0	3
		b	1	1
		c	2	1
		d	2	0
$G^{3}_{4,5}$		a	1	2
		b	1	1
		c	1	2
		d	2	0
$G^{4}_{4,5}$		a	0	2
		b	1	2
		c	1	1
		d	3	0
$G^{5}_{4,5}$		a	0	2
		b	2	1
		c	1	1

	d c	d 2 1
$G^{6}_{4,5}$		In out a 0 2 b 1 2 c 2 1 d 2 0
$G^{7}_{4,5}$		In out a 0 3 b 1 1 c 3 0 d 1 1
$G^{8}_{4,5}$		In out a 0 2 b 2 1 c 3 0 d 0 2
$G^{9}_{4,5}$		In out a 0 3 b 1 2 c 2 0 d 2 0

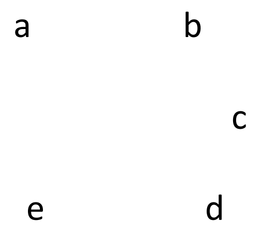
$G_{4,6}$ (7

$G^{1}_{4,6}$		In out a 1 2 b 1 2 c 2 1 d 2 1
$G^{2}_{4,6}$		In out a 0 3 b 1 2 c 2 1 d 3 0
$G^{3}_{4,6}$		In out a 0 3 b 2 1 c 2 1 d 2 1
		In out

$G^4_{4,6}$		a	1	2
		b	1	2
		c	1	2
		d	3	0

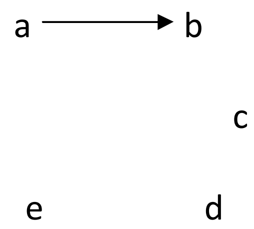
(6-3-2)البيانات المولدة بالمجموعة 5 (G_5) :

1) $G_{5,0}$:

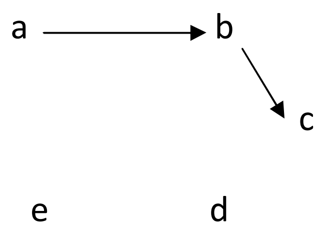
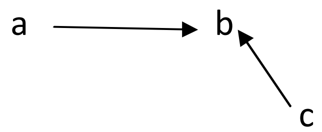
			In	out
$G^1_{5,0}$		a	0	0
		b	0	0
		c	0	0
		d	0	0
		e	0	0

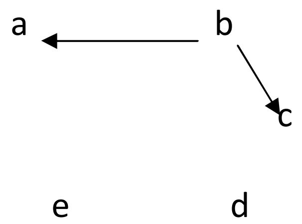
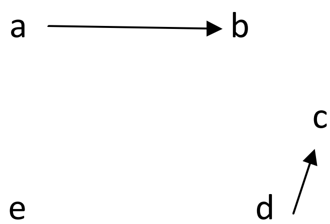
2

2) $G_{5,1}$:

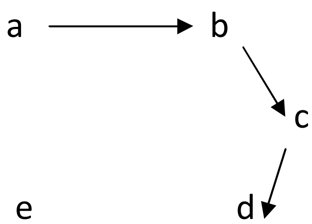
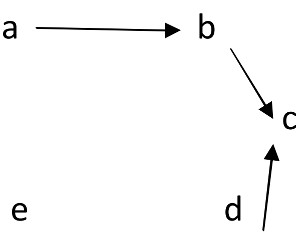
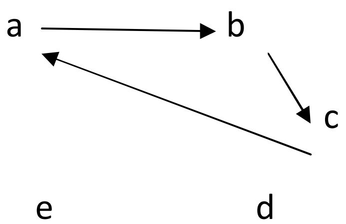
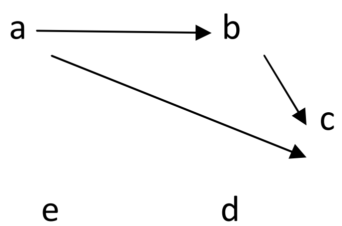
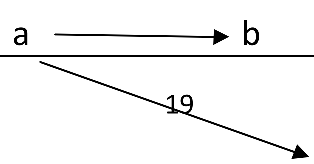
			In	out
$G^1_{5,1}$		a	0	1
		b	1	0
		c	0	0
		d	0	0
		e	0	0

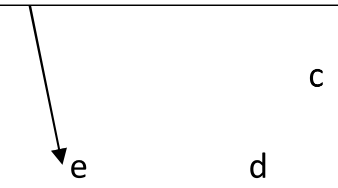
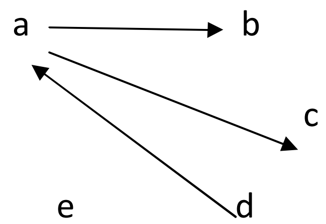
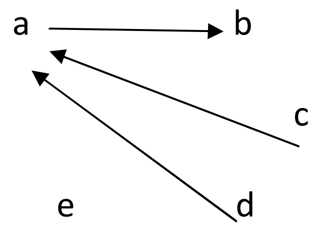
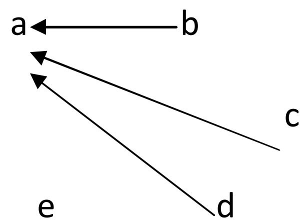
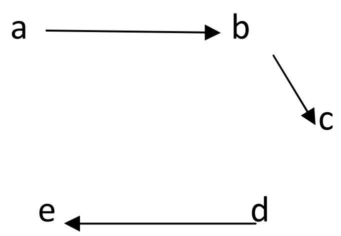
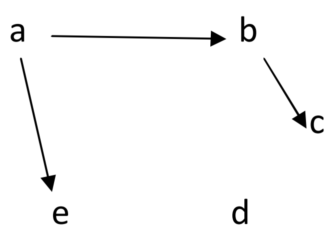
3) $G_{5,2}$:

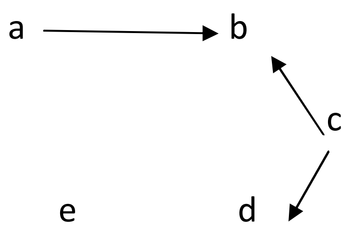
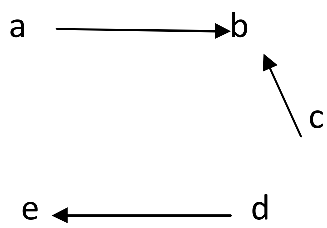
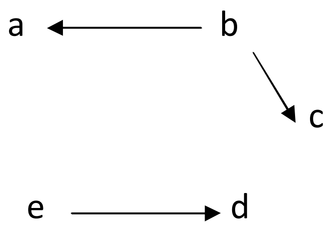
			In	out
$G^1_{5,2}$		a	0	1
		b	1	1
		c	1	0
		d	0	0
		e	0	0
			In	out
$G^2_{5,2}$		a	0	1
		b	2	0
		c	0	1

		d	0	0
	e d	e	0	0
$G^3_{5,2}$			In	out
		a	1	0
		b	0	2
		c	1	0
		d	0	0
		e	0	0
$G^4_{5,2}$			In	out
		a	0	1
		b	1	0
		c	1	0
		d	0	1
		e	0	0

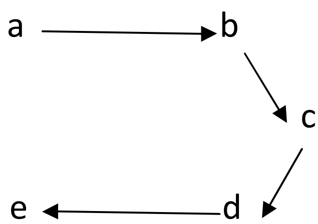
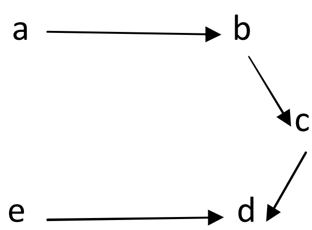
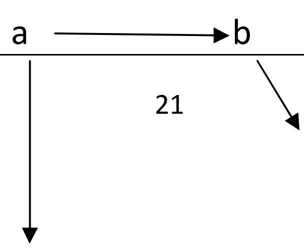
: $G_{5,3}$ (4

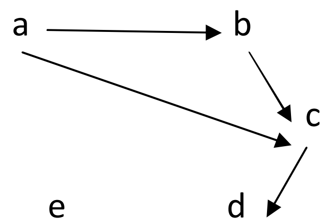
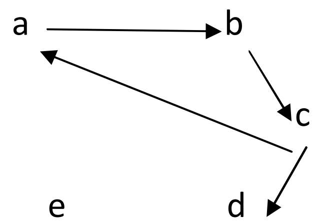
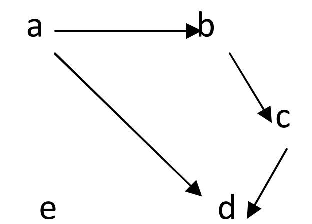
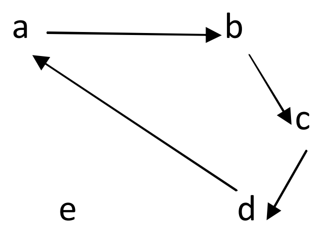
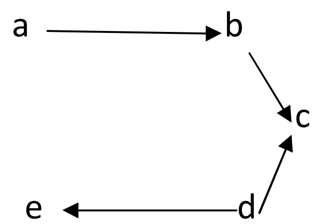
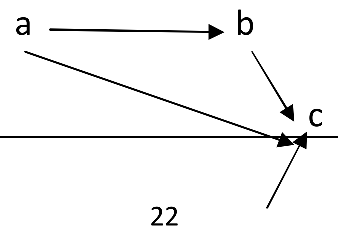
$G^1_{5,3}$			In	out
		a	0	1
		b	1	1
		c	1	1
		d	1	0
		e	0	0
$G^2_{5,3}$			In	out
		a	0	1
		b	1	1
		c	2	0
		d	0	1
		e	0	0
$G^3_{5,3}$			In	out
		a	1	1
		b	1	1
		c	1	1
		d	0	0
		e	0	0
$G^4_{5,3}$			In	out
		a	0	2
		b	1	1
		c	2	0
		d	0	0
		e	0	0
			In	out
		a	0	3

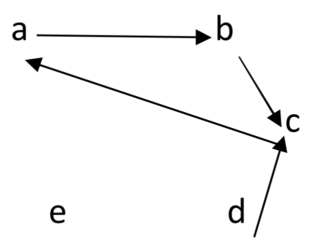
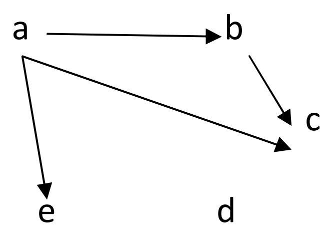
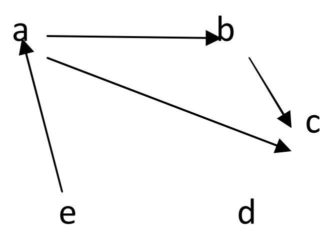
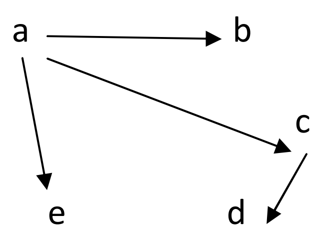
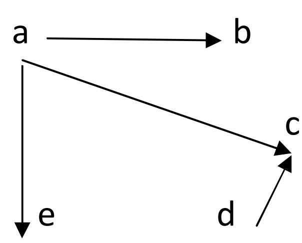
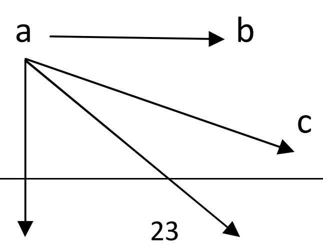
$G^{5}_{5,3}$		b 1 0 c 1 0 d 1 0 e 0 0
$G^{6}_{5,3}$		In out a 1 2 b 1 0 c 1 0 d 0 1 e 0 0
$G^{7}_{5,3}$		In out a 2 1 b 1 0 c 0 1 d 0 1 e 0 0
$G^{8}_{5,3}$		In out a 3 0 b 0 1 c 0 1 d 0 1 e 0 0
$G^{9}_{5,3}$		In out a 0 1 b 1 1 c 1 0 d 0 1 e 1 0
$G^{10}_{5,3}$		In out a 0 2 b 1 1 c 1 0 d 0 0 e 1 0

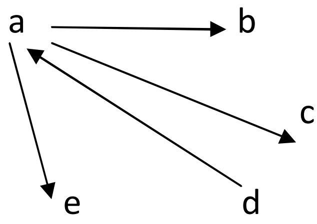
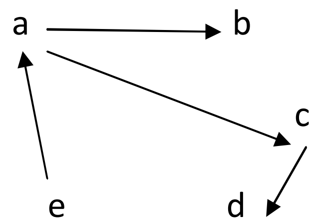
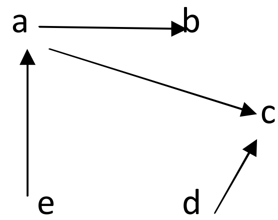
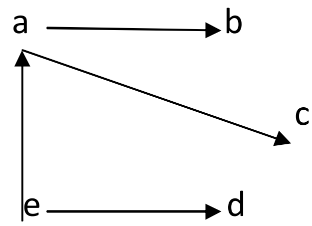
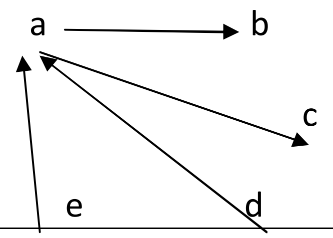
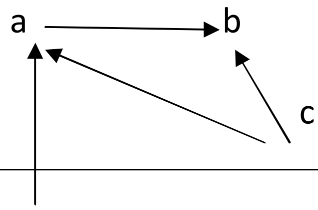
$G^{11}_{5,3}$		In out a 0 1 b 2 0 c 0 2 d 1 0 e 0 0
$G^{12}_{5,3}$		In out a 0 1 b 2 0 c 0 1 d 0 1 e 1 0
$G^{13}_{5,3}$		In out a 1 0 b 0 2 c 1 0 d 1 0 e 0 1

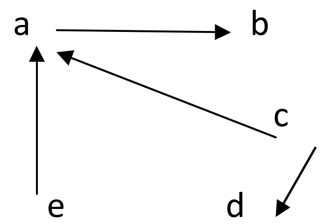
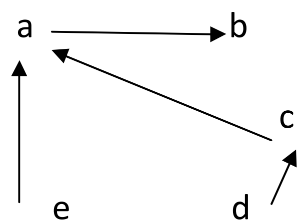
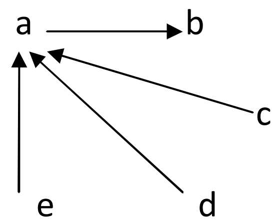
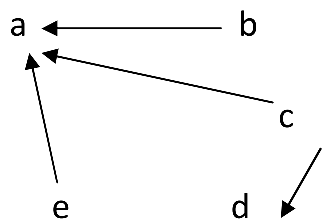
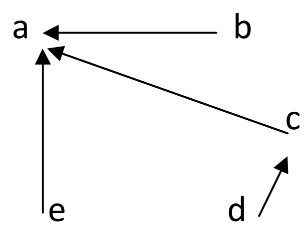
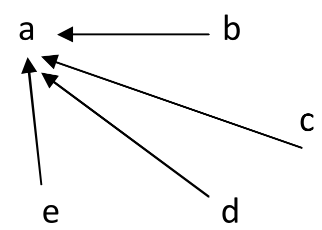
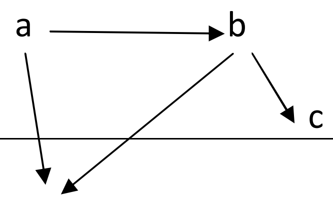
$G_{5,4}$ (5

$G^{1}_{5,4}$		In out a 0 1 b 1 1 c 1 1 d 1 1 e 1 0
$G^{2}_{5,4}$		In out a 0 1 b 1 1 c 1 1 d 2 0 e 0 1
		In out a 0 2

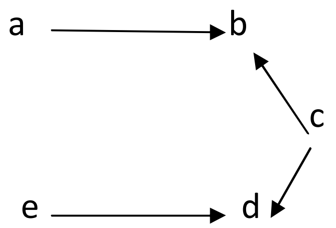
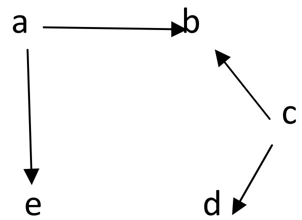
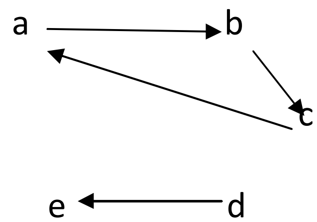
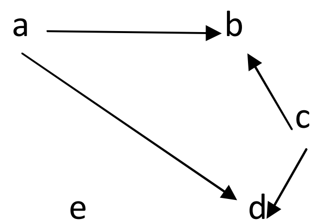
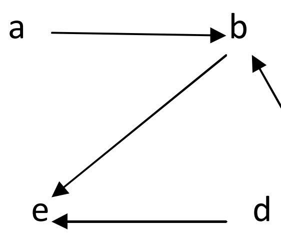
$G^3_{5,4}$		b 1 1 c 1 1 d 1 0 e 1 0
$G^4_{5,4}$		In out a 0 2 b 1 1 c 2 1 d 1 0 e 0 0
$G^5_{5,4}$		In out a 1 1 b 1 1 c 1 2 d 1 0 e 0 0
$G^6_{5,4}$		In out a 0 2 b 1 1 c 1 1 d 2 0 e 0 0
$G^7_{5,4}$		In out a 1 1 b 1 1 c 1 1 d 1 1 e 0 0
$G^8_{5,4}$		In out a 0 1 b 1 1 c 2 0 d 0 2 e 1 0
$G^9_{5,4}$		In out a 0 2 b 1 1 c 3 0

	e d	d 0 1 e 0 0
$G^{10}_{5,4}$		In out a 1 1 b 1 1 c 2 1 d 0 1 e 0 0
$G^{11}_{5,4}$		In out a 0 3 b 1 1 c 2 0 d 0 0 e 1 0
$G^{12}_{5,4}$		In out a 1 2 b 1 1 c 2 0 d 0 0 e 0 1
$G^{13}_{5,4}$		In out a 0 3 b 1 0 c 1 1 d 1 0 e 1 0
$G^{14}_{5,4}$		In out a 0 3 b 1 0 c 2 0 d 0 1 e 1 0
$G^{15}_{5,4}$		In out a 0 4 b 1 0 c 1 0

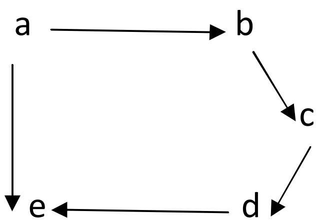
	e d	d 1 0 e 1 0
$G^{16}_{5,4}$		In out a 1 3 b 1 0 c 1 0 d 0 1 e 1 0
$G^{17}_{5,4}$		In out a 1 2 b 1 0 c 1 1 d 1 0 e 0 1
$G^{18}_{5,4}$		In out a 1 2 b 1 0 c 2 0 d 0 1 e 0 1
$G^{19}_{5,4}$		In out a 1 2 b 1 0 c 1 0 d 1 0 e 0 2
$G^{20}_{5,4}$		In out a 2 2 b 1 0 c 1 0 d 0 1 e 0 1
$G^{21}_{5,4}$		In out a 2 1 b 2 0 c 0 2

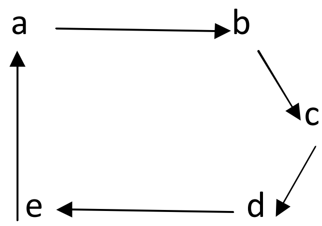
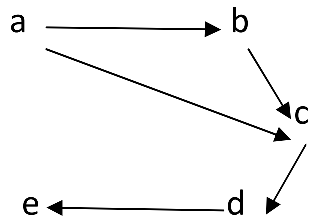
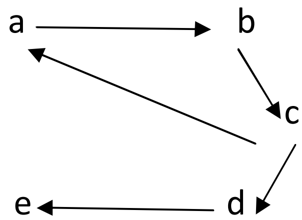
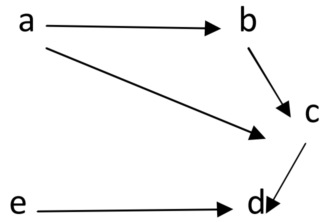
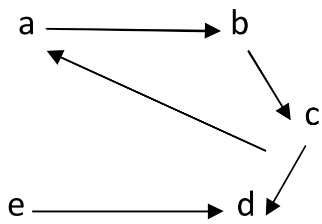
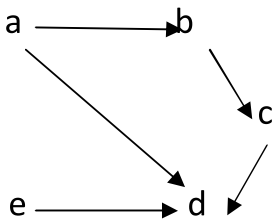
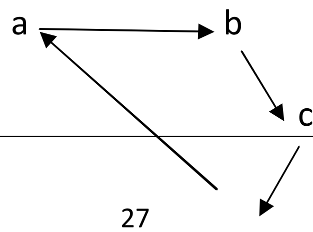
Graph	Diagram	Vertex	In	Out
	e d	d	0	0
		e	0	1
$G^{22}_{5,4}$		a	2	1
		b	1	0
		c	0	2
		d	1	0
		e	0	1
$G^{23}_{5,4}$		a	2	1
		b	1	0
		c	1	1
		d	0	1
		e	0	1
$G^{24}_{5,4}$		a	3	1
		b	1	0
		c	0	1
		d	0	1
		e	0	1
$G^{25}_{5,4}$		a	3	0
		b	0	1
		c	0	2
		d	1	0
		e	0	1
$G^{26}_{5,4}$		a	3	0
		b	0	1
		c	1	1
		d	0	1
		e	0	1
$G^{27}_{5,4}$		a	4	0
		b	0	1
		c	0	1
		d	0	1
		e	0	1
$G^{28}_{5,4}$		a	0	2
		b	1	2
		c	1	0

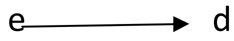
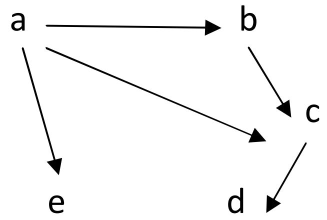
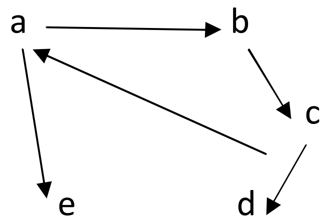
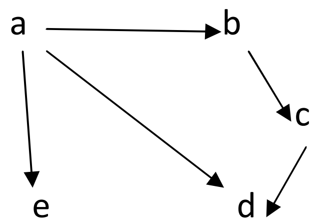
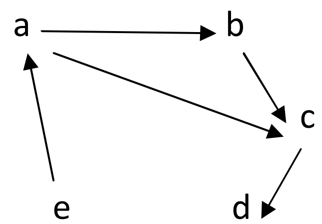
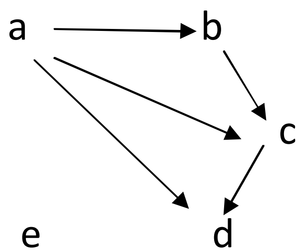
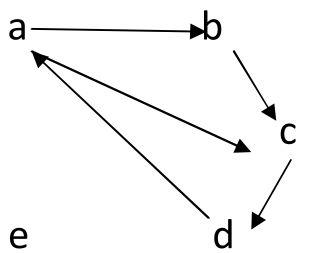
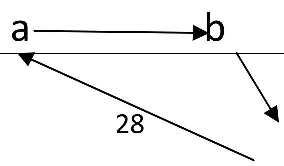
: 5

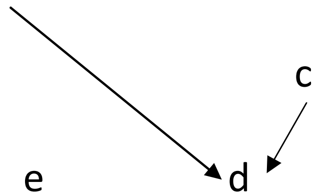
	e d	d 0 0 e 2 0
$G^{29}_{5,4}$		In out a 0 1 b 2 0 c 0 2 d 2 0 e 0 1
$G^{30}_{5,4}$		In out a 0 2 b 2 0 c 0 2 d 1 0 e 1 0
$G^{31}_{5,4}$		In out a 1 1 b 1 1 c 1 1 d 0 1 e 1 0
$G^{32}_{5,4}$		In out a 0 2 b 2 0 c 0 2 d 2 0 e 0 0
$G^{33}_{5,4}$		In out a 0 1 b 2 1 c 0 1 d 0 1 e 2 0

$G_{5,5}$ (6

$G^{1}_{5,5}$		In out a 0 2 b 1 1 c 1 1 d 1 1 e 2 0
		In out

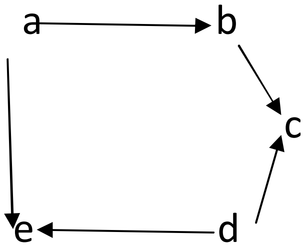
$G^2_{5,5}$		a 1 1 b 1 1 c 1 1 d 1 1 e 1 1
$G^3_{5,5}$		In out a 0 2 b 1 1 c 2 1 d 1 1 e 1 0
$G^4_{5,5}$		In out a 1 1 b 1 1 c 1 2 d 1 1 e 1 0
$G^5_{5,5}$		In out a 0 2 b 1 1 c 2 1 d 2 0 e 0 1
$G^6_{5,5}$		In out a 1 1 b 1 1 c 1 2 d 2 0 e 0 1
$G^7_{5,5}$		In out a 0 2 b 1 1 c 1 1 d 3 0 e 0 1
$G^8_{5,5}$		In out a 1 1 b 1 1 c 1 1

		d 2 1 e 0 1
$G^{9}_{5,5}$		In out a 0 3 b 1 1 c 2 1 d 1 0 e 1 0
$G^{10}_{5,5}$		In out a 1 2 b 1 1 c 1 2 d 1 0 e 1 0
$G^{11}_{5,5}$		In out a 0 3 b 1 1 c 1 1 d 2 0 e 1 0
$G^{12}_{5,5}$		In out a 1 2 b 1 1 c 2 1 d 1 0 e 0 1
$G^{13}_{5,5}$		In out a 0 3 b 1 1 c 2 1 d 2 0 e 0 0
$G^{14}_{5,5}$		In out a 1 2 b 1 1 c 2 1 d 1 1 e 0 0
$G^{15}_{5,5}$		In out a 1 2



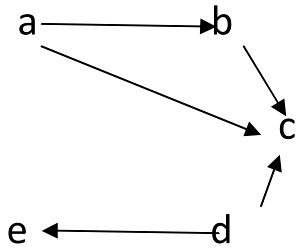
	In	out
b	1	1
c	1	2
d	2	0
e	0	0

$G^{16}_{5,5}$



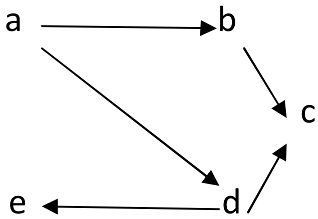
	In	out
a	0	2
b	1	1
c	2	0
d	0	2
e	2	0

$G^{17}_{5,5}$



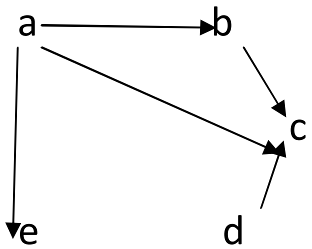
	In	out
a	0	2
b	1	1
c	3	0
d	0	2
e	1	0

$G^{18}_{5,5}$



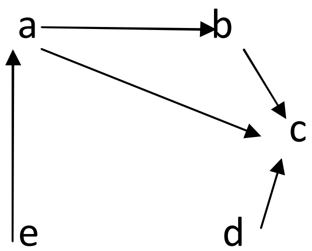
	In	out
a	0	2
b	1	1
c	2	0
d	1	2
e	1	0

$G^{19}_{5,5}$



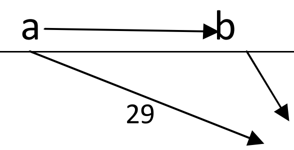
	In	out
a	0	3
b	1	1
c	3	0
d	0	1
e	1	0

$G^{20}_{5,5}$

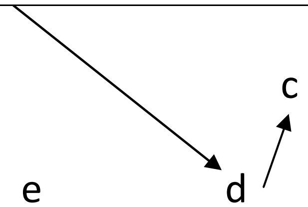
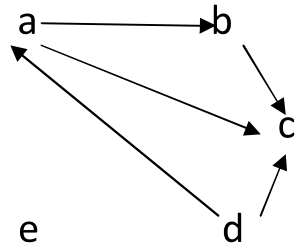
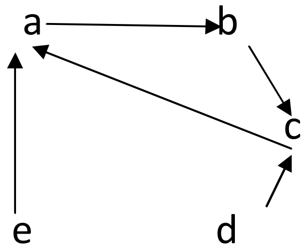
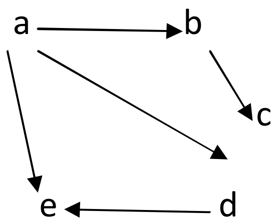
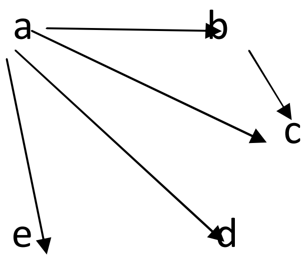
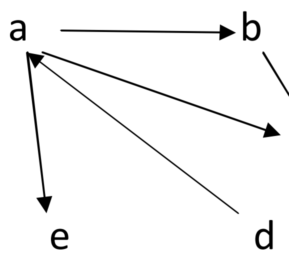
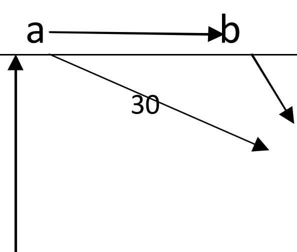


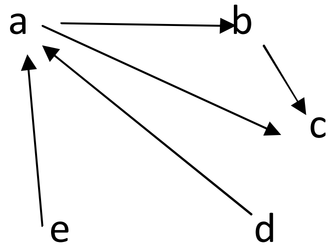
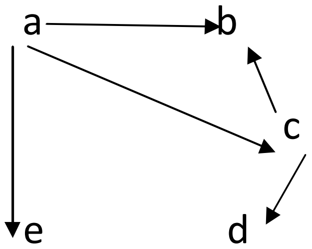
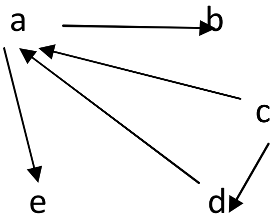
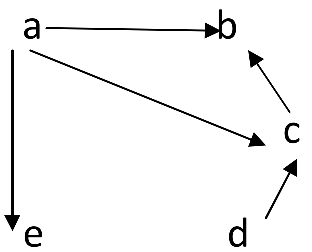
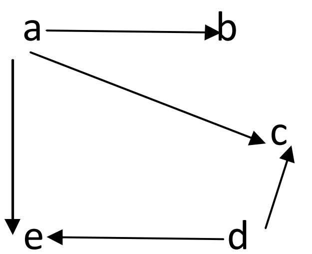
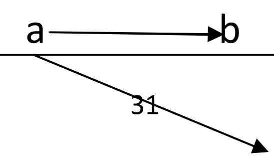
	In	out
a	1	2
b	1	1
c	3	0
d	0	1
e	0	1

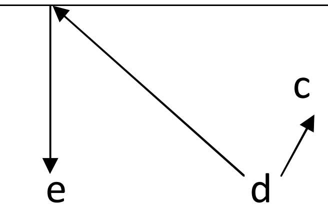
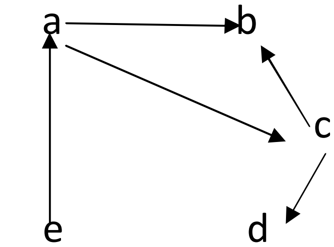
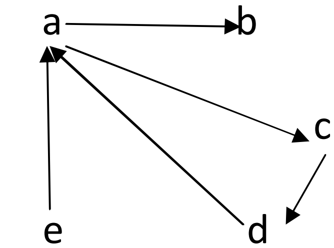
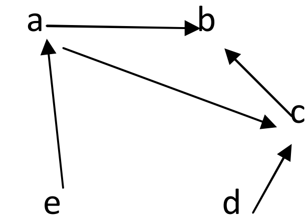
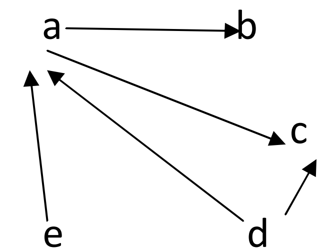
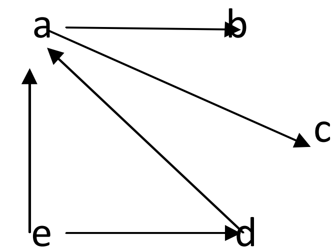
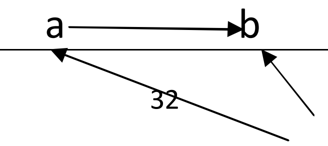
$G^{21}_{5,5}$

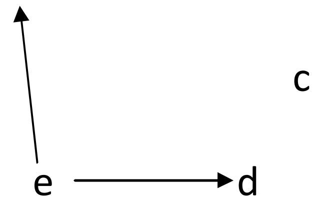
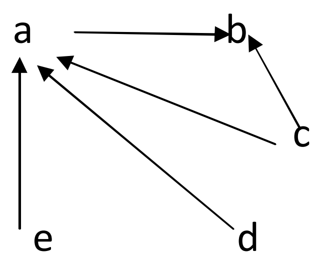
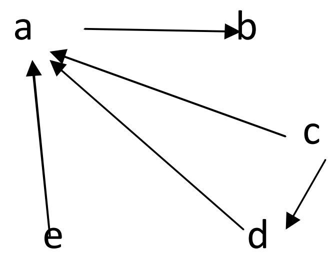
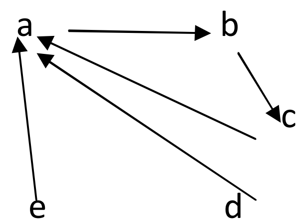
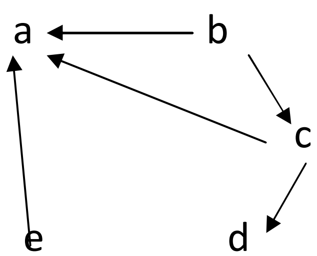
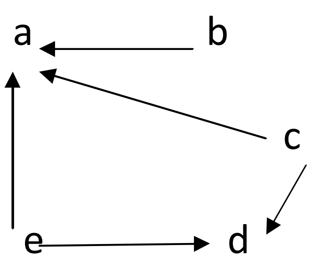
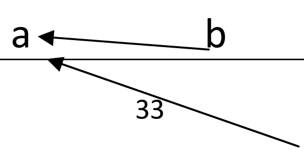


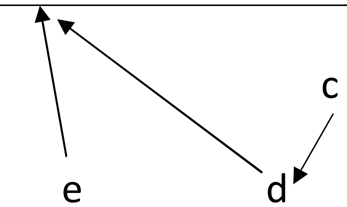
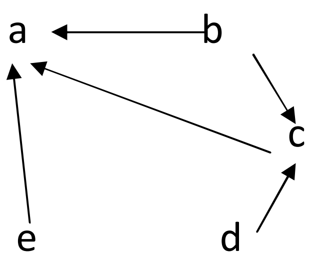
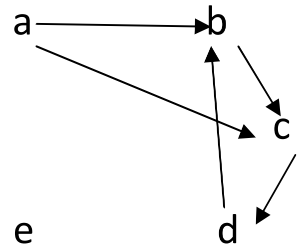
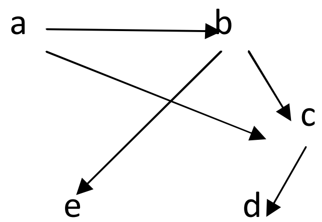
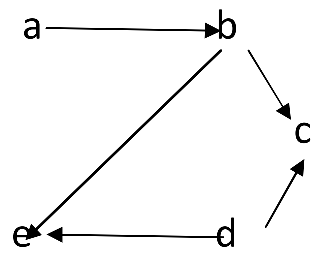
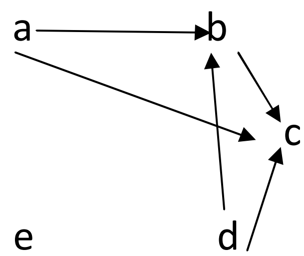
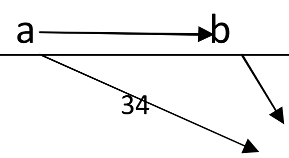
	In	out
a	0	3

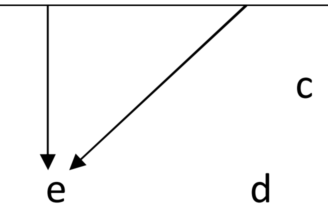
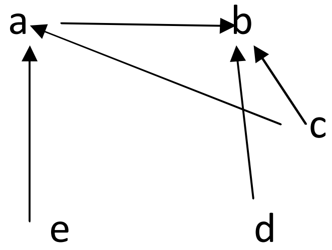
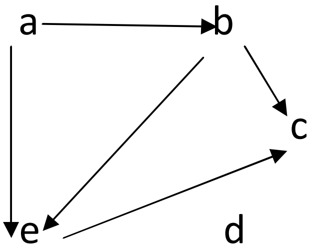
Graph	Digraph	Vertex	In	out
		b	1	1
		c	3	0
		d	1	1
		e	0	0
$G^{22}_{5,5}$		a	1	2
		b	1	1
		c	3	0
		d	0	2
		e	0	0
$G^{23}_{5,5}$		a	2	1
		b	1	1
		c	2	1
		d	0	1
		e	0	1
$G^{24}_{5,5}$		a	0	3
		b	1	1
		c	2	0
		d	0	1
		e	2	0
$G^{25}_{5,5}$		a	0	4
		b	1	1
		c	2	0
		d	1	0
		e	1	0
$G^{26}_{5,5}$		a	1	3
		b	1	1
		c	2	0
		d	0	1
		e	1	0
$G^{27}_{5,5}$		a	1	2

		b 1 1 c 3 0 d 0 1 e 0 1
$G^{28}_{5,5}$		In out a 2 2 b 1 1 c 2 0 d 0 1 e 0 1
$G^{29}_{5,5}$		In out a 0 3 b 2 0 c 1 2 d 1 0 e 1 0
$G^{30}_{5,5}$		In out a 1 3 b 1 0 c 1 1 d 1 1 e 1 0
$G^{31}_{5,5}$		In out a 0 3 b 2 0 c 2 1 d 0 1 e 1 0
$G^{32}_{5,5}$		In out a 0 3 b 1 0 c 2 0 d 0 2 e 2 0
$G^{33}_{5,5}$		In out a 1 3

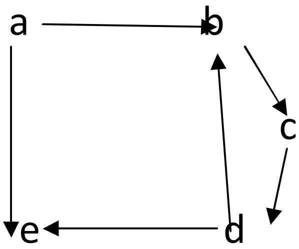
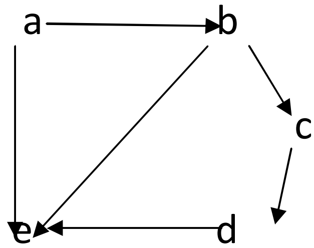
		b 1 0 c 2 0 d 0 2 e 1 0
$G^{34}_{5,5}$		In out a 1 2 b 2 0 c 1 2 d 1 0 e 0 1
$G^{35}_{5,5}$		In out a 2 2 b 1 0 c 1 1 d 1 1 e 0 1
$G^{36}_{5,5}$		In out a 1 2 b 2 0 c 2 1 d 0 1 e 0 1
$G^{37}_{5,5}$		In out a 2 2 b 1 0 c 2 0 d 0 2 e 0 1
$G^{38}_{5,5}$		In out a 2 2 b 1 0 c 1 0 d 1 1 e 0 2
$G^{39}_{5,5}$		In out a 2 1

		b 2 0 c 0 2 d 1 0 e 0 2
$G^{40}_{5,5}$		In out a 3 1 b 2 0 c 0 2 d 0 1 e 0 1
$G^{41}_{5,5}$		In out a 3 1 b 1 0 c 0 2 d 1 1 e 0 1
$G^{42}_{5,5}$		In out a 3 1 b 1 1 c 1 1 d 0 1 e 0 1
$G^{43}_{5,5}$		In out a 3 0 b 0 2 c 1 2 d 1 0 e 0 1
$G^{44}_{5,5}$		In out a 3 0 b 0 1 c 0 2 d 2 0 e 0 2
$G^{45}_{5,5}$		In out a 4 0

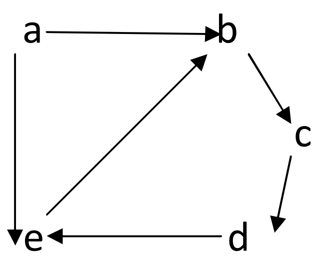
		b 0 1 c 0 2 d 1 1 e 0 1
$G^{46}_{5,5}$		In out a 3 0 b 0 2 c 2 1 d 0 1 e 0 1
$G^{47}_{5,5}$		In out a 0 2 b 2 1 c 2 1 d 1 1 e 0 0
$G^{48}_{5,5}$		In out a 0 2 b 1 2 c 2 1 d 1 0 e 1 0
$G^{49}_{5,5}$		In out a 0 1 b 1 2 c 2 0 d 0 2 e 2 0
$G^{50}_{5,5}$		In out a 0 2 b 2 1 c 3 0 d 0 2 e 0 0
$G^{51}_{5,5}$		In out a 0 3

		 b c d e	 1 2 0 2	 2 0 0 0
$G^{52}_{5,5}$		 a b c d e	In 2 2 0 1 0	out 1 1 2 0 1
$G^{53}_{5,5}$		 a b c d e	In 0 1 2 0 2	out 2 2 0 0 1

$G_{5,6}$(7

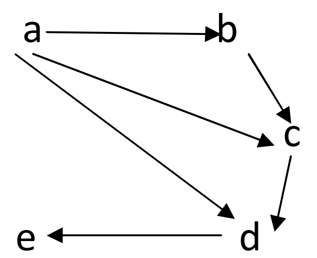
$G^{1}_{5,6}$		 a b c d e	In 0 2 1 1 2	out 2 1 1 2 0
$G^{2}_{5,6}$		 a b c d e	In 0 1 1 1 3	out 2 2 1 1 0

$G^3_{5,6}$



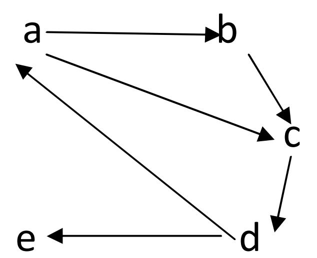
	In	out
a	0	2
b	2	1
c	1	1
d	1	1
e	2	1

$G^4_{5,6}$



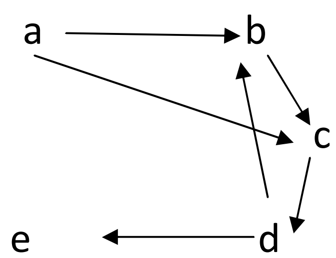
	In	out
a	0	3
b	1	1
c	2	1
d	2	1
e	1	0

$G^5_{5,6}$



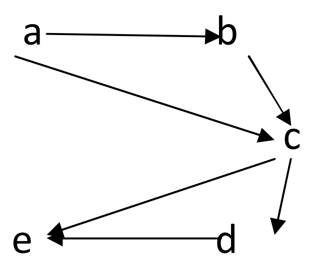
	In	out
a	1	2
b	1	1
c	2	1
d	1	2
e	1	0

$G^6_{5,6}$



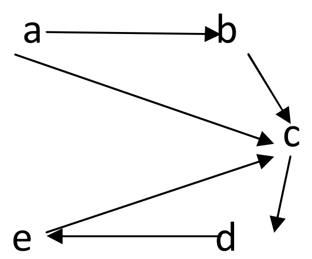
	In	out
a	0	2
b	2	1
c	2	1
d	1	2
e	1	0

$G^7_{5,6}$



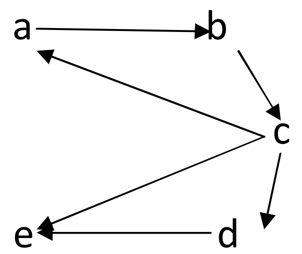
	In	out
a	0	2
b	1	1
c	2	2
d	1	1
e	2	0

$G^{8}_{5,6}$



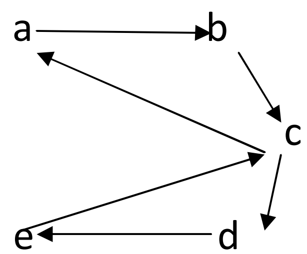
	In	out
a	0	2
b	1	1
c	3	1
d	1	1
e	1	1

$G^{9}_{5,6}$



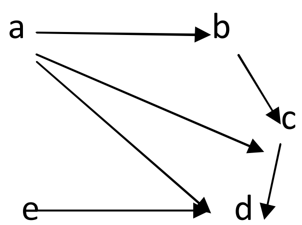
	In	out
a	1	1
b	1	1
c	1	3
d	1	1
e	2	0

$G^{10}_{5,6}$



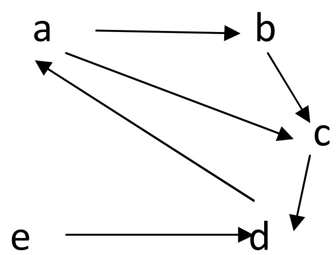
	In	out
a	1	1
b	1	1
c	2	2
d	1	1
e	1	1

$G^{11}_{5,6}$



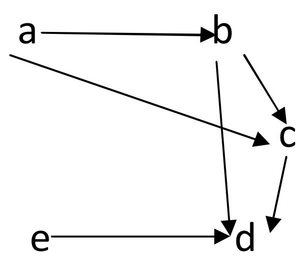
	In	out
a	0	3
b	1	1
c	2	1
d	3	0
e	0	1

$G^{12}_{5,6}$

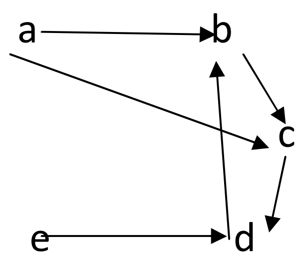
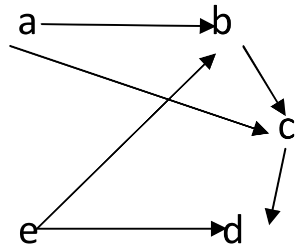
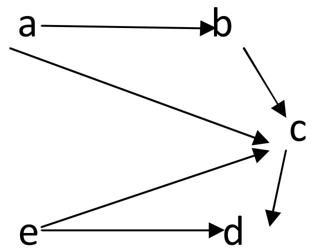
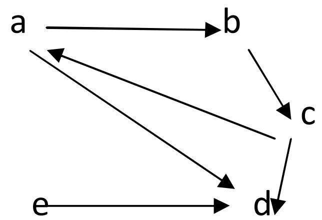
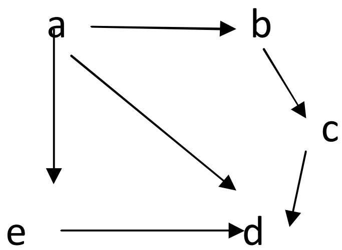
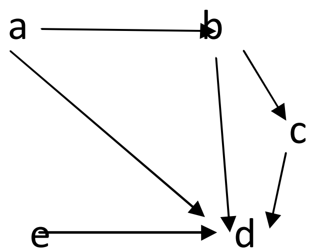


	In	out
a	1	2
b	1	1
c	2	1
d	2	1
e	0	1

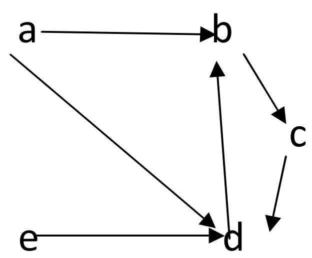
$G^{13}_{5,6}$



	In	out
a	0	2
b	1	2
c	2	1
d	3	0
e	0	1

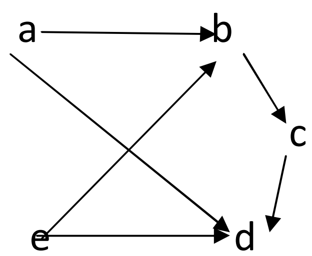
Graph	Diagram	Vertex	In	out
$G^{14}_{5,6}$		a	0	2
		b	2	1
		c	2	1
		d	2	1
		e	0	1
$G^{15}_{5,6}$		a	0	2
		b	2	1
		c	2	1
		d	2	0
		e	0	2
$G^{16}_{5,6}$		a	0	2
		b	1	1
		c	3	1
		d	2	0
		e	0	2
$G^{17}_{5,6}$		a	1	2
		b	1	1
		c	1	2
		d	3	0
		e	0	1
$G^{18}_{5,6}$		a	0	3
		b	1	1
		c	1	1
		d	3	0
		e	1	1
$G^{19}_{5,6}$		a	0	2
		b	1	2
		c	1	1
		d	4	0
		e	0	1

$G^{20}_{5,6}$



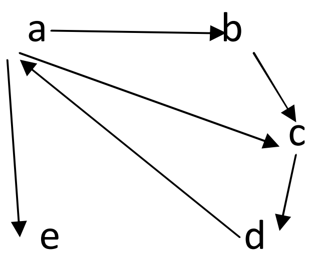
	In	out
a	0	2
b	2	1
c	1	1
d	3	1
e	0	1

$G^{21}_{5,6}$



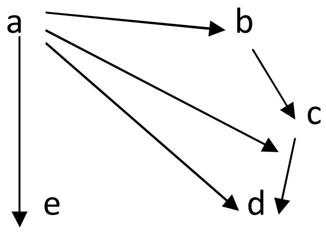
	In	out
a	0	2
b	2	1
c	1	1
d	3	0
e	0	2

$G^{22}_{5,6}$



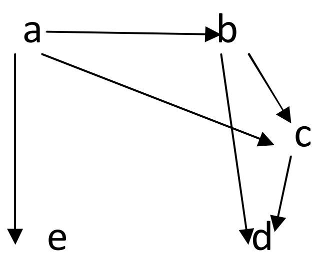
	In	out
a	1	3
b	1	1
c	2	1
d	1	1
e	1	0

$G^{23}_{5,6}$



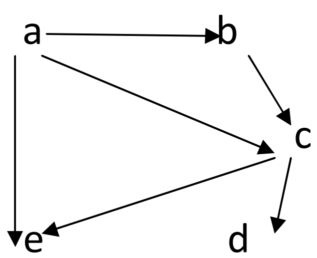
	In	out
a	0	4
b	1	1
c	2	1
d	2	0
e	1	0

$G^{24}_{5,6}$



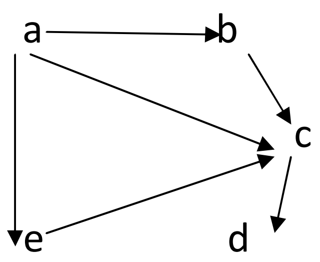
	In	out
a	0	3
b	1	2
c	2	1
d	2	0
e	1	0

$G^{25}_{5,6}$



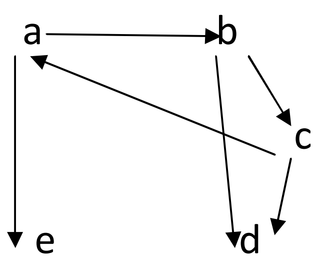
	In	out
a	0	3
b	1	1
c	2	2
d	1	0
e	2	0

$G^{26}_{5,6}$



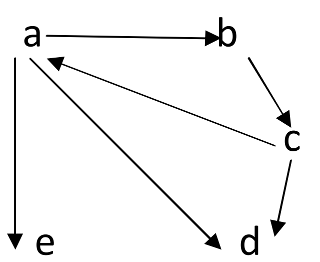
	In	out
a	0	3
b	1	1
c	3	1
d	1	0
e	1	1

$G^{27}_{5,6}$



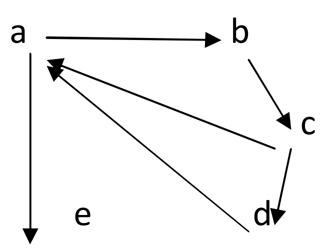
	In	out
a	1	2
b	1	2
c	1	2
d	2	0
e	1	0

$G^{28}_{5,6}$



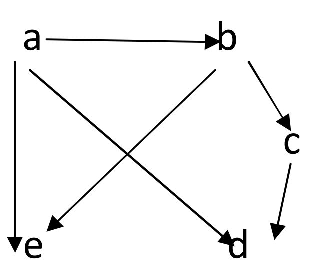
	In	out
a	1	3
b	1	1
c	1	2
d	2	0
e	1	0

$G^{29}_{5,6}$



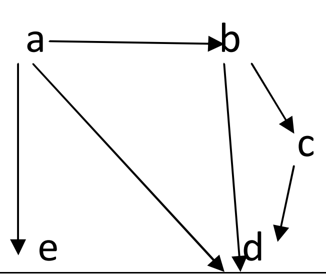
	In	out
a	2	2
b	1	1
c	1	2
d	1	1
e	1	0

$G^{30}_{5,6}$



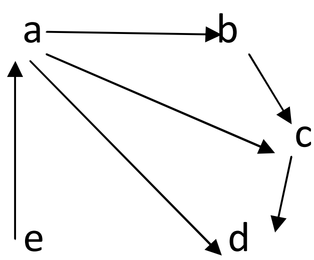
	In	out
a	0	3
b	1	2
c	1	1
d	2	0
e	2	0

$G^{31}_{5,6}$



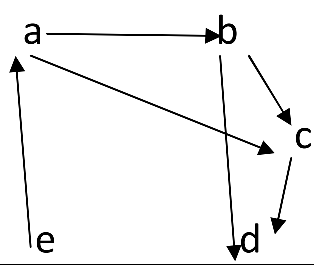
	In	out
a	0	3
b	1	2
c	1	1
d	3	0
e	1	0

$G^{32}_{5,6}$



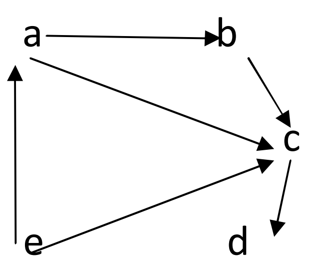
	In	out
a	1	3
b	1	1
c	2	1
d	2	0
e	0	1

$G^{33}_{5,6}$



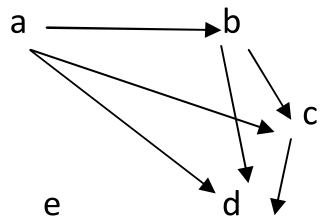
	In	out
a	1	2
b	1	2
c	2	1
d	2	0
e	0	1

$G^{34}_{5,6}$



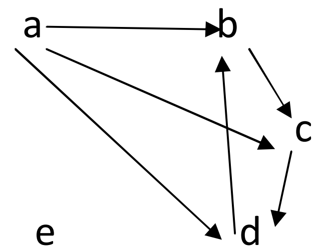
	In	out
a	1	3
b	1	1
c	3	1
d	1	0
e	0	2

$G^{35}_{5,6}$



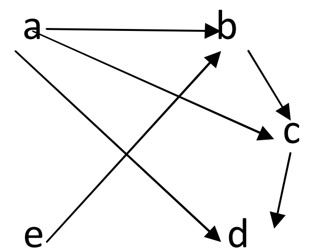
	In	out
a	0	3
b	1	2
c	2	1
d	3	0
e	0	0

$G^{36}_{5,6}$



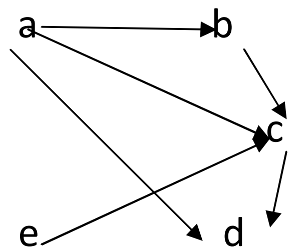
	In	out
a	0	3
b	2	1
c	2	1
d	2	1
e	0	0

$G^{37}_{5,6}$



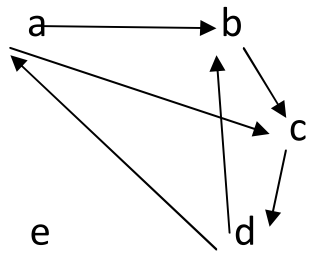
	In	out
a	0	3
b	2	1
c	2	1
d	2	0
e	0	1

$G^{38}_{5,6}$



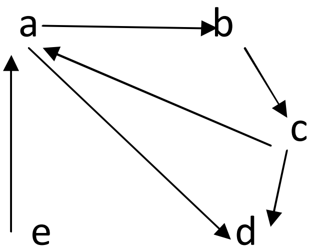
	In	out
a	0	3
b	1	1
c	3	1
d	2	0
e	0	1

$G^{39}_{5,6}$



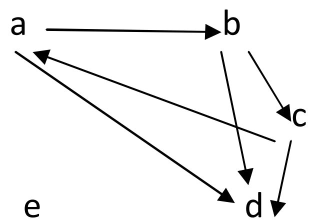
	In	out
a	1	2
b	2	1
c	2	1
d	1	2
e	0	0

$G^{40}_{5,6}$



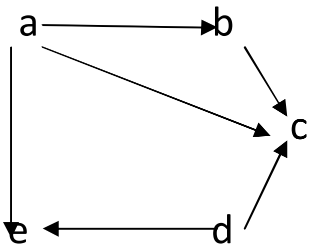
	In	out
a	2	2
b	1	1
c	1	2
d	2	0
e	0	1

$G^{41}_{5,6}$



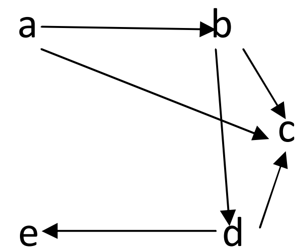
	In	out
a	1	2
b	1	2
c	1	2
d	3	0
e	0	0

$G^{42}_{5,6}$



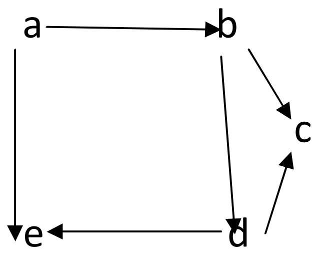
	In	out
a	0	3
b	1	1
c	3	0
d	0	2
e	2	0

$G^{43}_{5,6}$



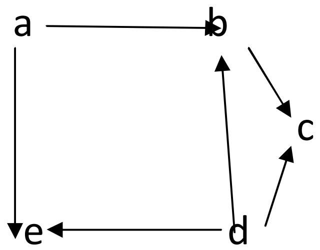
	In	out
a	0	2
b	1	2
c	3	0
d	1	2
e	1	0

$G^{44}_{5,6}$



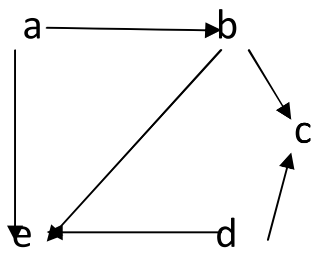
	In	out
a	0	2
b	1	2
c	2	0
d	1	2
e	2	0

$G^{45}_{5,6}$



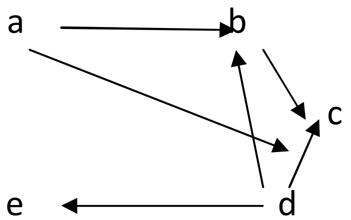
	In	out
a	0	2
b	2	1
c	2	0
d	0	3
e	2	0

$G^{46}_{5,6}$



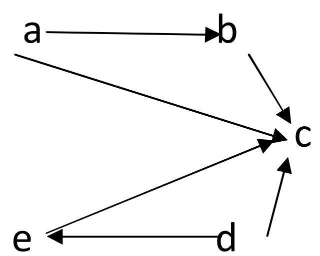
	In	out
a	0	2
b	1	2
c	2	0
d	0	2
e	3	0

$G^{47}_{5,6}$



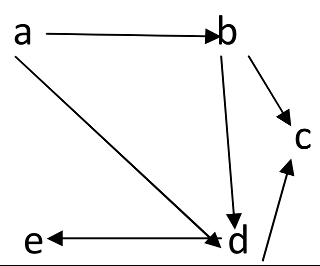
	In	out
a	0	2
b	2	1
c	3	0
d	0	3
e	1	0

$G^{48}_{5,6}$



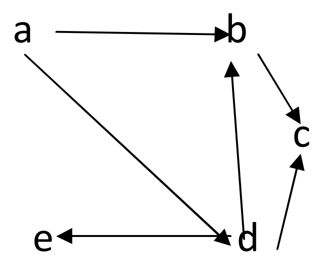
	In	out
a	0	2
b	1	1
c	4	0
d	0	2
e	1	1

$G^{49}_{5,6}$



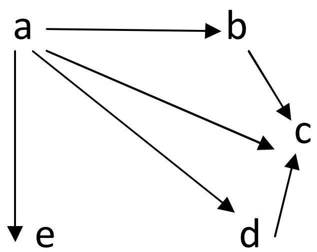
	In	out
a	0	2
b	1	2
c	2	0
d	2	2
e	1	0

$G^{50}_{5,6}$



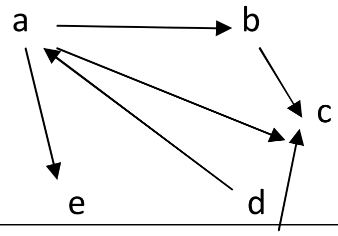
	In	out
a	0	2
b	2	1
c	2	0
d	1	3
e	1	0

$G^{51}_{5,6}$



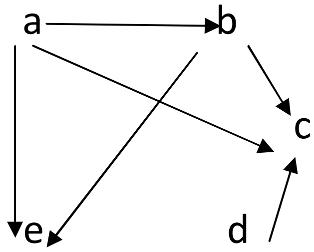
	In	out
a	0	4
b	1	1
c	3	0
d	1	1
e	1	0

$G^{52}_{5,6}$



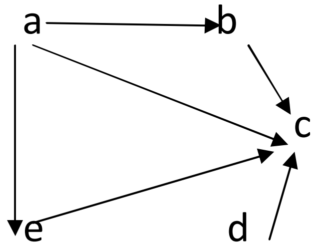
	In	out
a	1	3
b	1	1
c	3	0
d	0	2
e	1	0

$G^{53}_{5,6}$



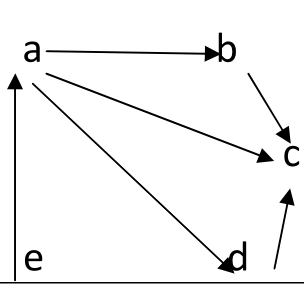
	In	out
a	0	3
b	1	2
c	3	0
d	0	1
e	2	0

$G^{54}_{5,6}$

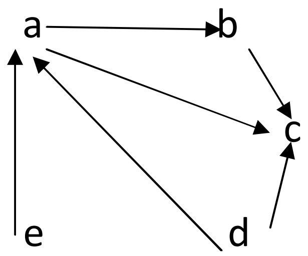
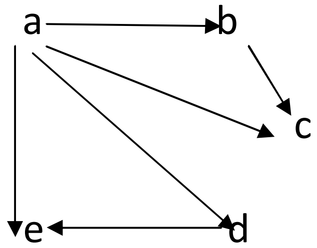
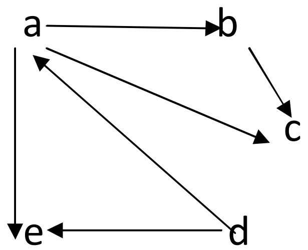
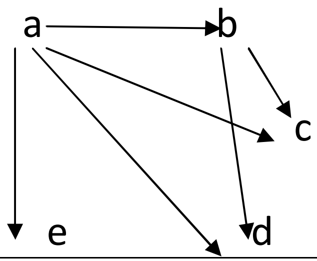
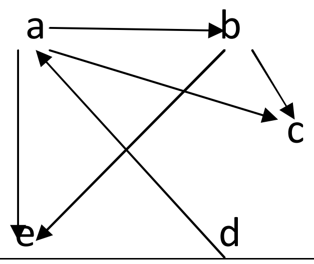
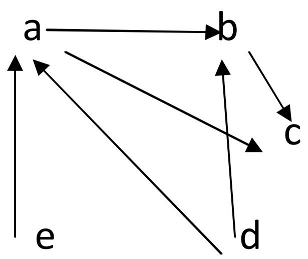


	In	out
a	0	3
b	1	1
c	4	0
d	0	1
e	1	1

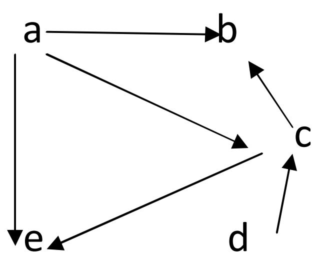
$G^{55}_{5,6}$



	In	out
a	1	3
b	1	1
c	3	0
d	1	1
e	0	1

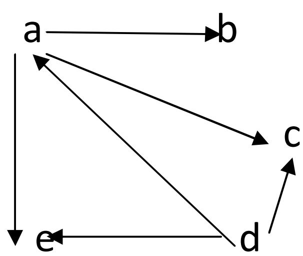
$G^{56}_{5,6}$		In out a 2 2 b 1 1 c 3 0 d 0 2 e 0 1
$G^{57}_{5,6}$		In out a 0 4 b 1 1 c 2 0 d 1 1 e 2 0
$G^{58}_{5,6}$		In out a 1 3 b 1 1 c 2 0 d 0 2 e 2 0
$G^{59}_{5,6}$		In out a 0 4 b 1 2 c 2 0 d 2 0 e 1 0
$G^{60}_{5,6}$		In out a 1 3 b 1 2 c 2 0 d 0 1 e 2 0
$G^{61}_{5,6}$		In out a 2 2 b 2 1 c 2 0 d 0 2 e 0 1

$G^{62}_{5,6}$



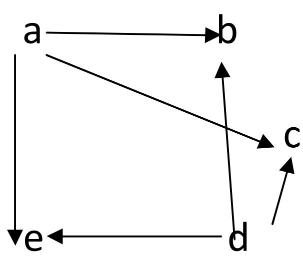
	In	out
a	0	3
b	2	0
c	2	2
d	0	1
e	2	0

$G^{63}_{5,6}$



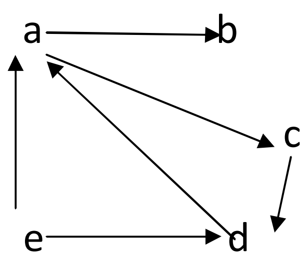
	In	out
a	1	3
b	1	0
c	2	0
d	0	3
e	2	0

$G^{64}_{5,6}$



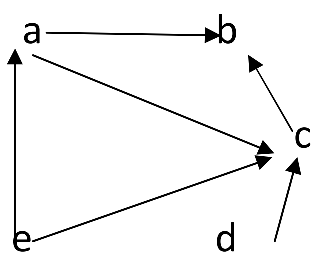
	In	out
a	0	3
b	2	0
c	2	0
d	0	3
e	2	0

$G^{65}_{5,6}$



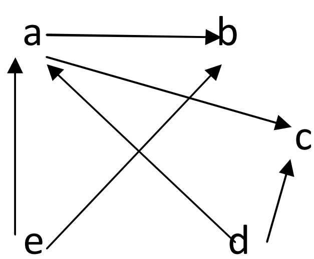
	In	out
a	2	2
b	1	0
c	1	1
d	2	1
e	0	2

$G^{66}_{5,6}$

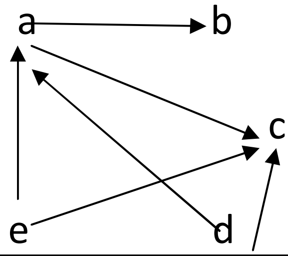
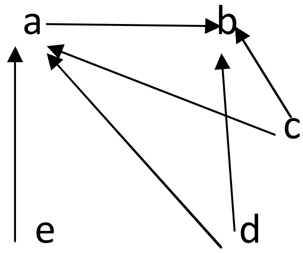
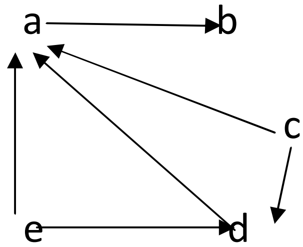
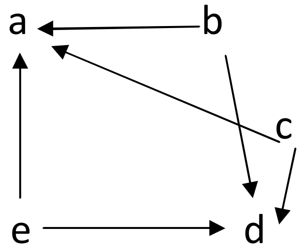
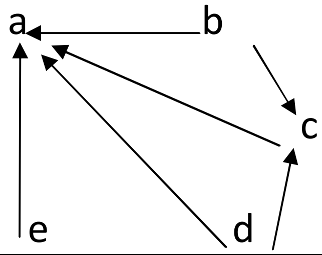


	In	out
a	1	2
b	2	0
c	3	1
d	0	1
e	0	2

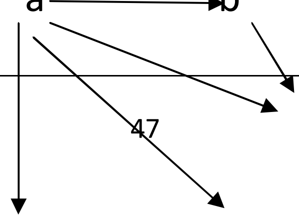
$G^{67}_{5,6}$

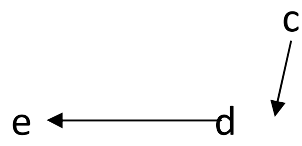
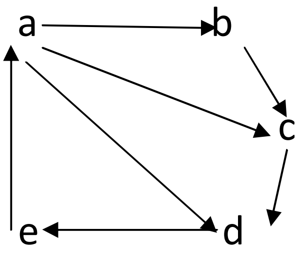
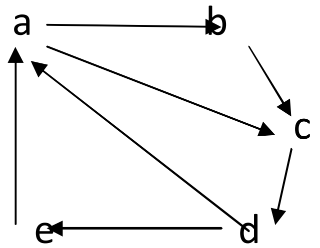
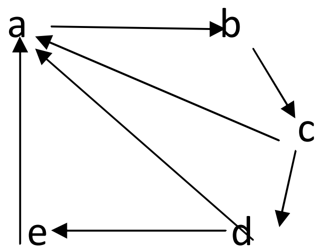
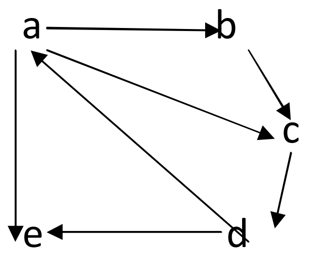
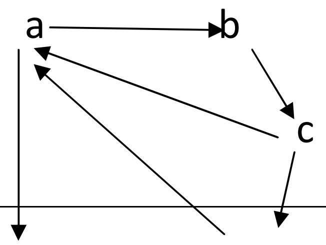


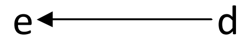
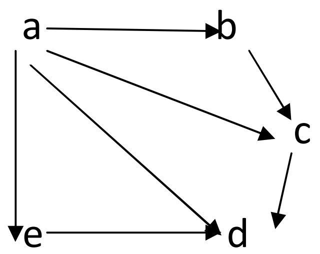
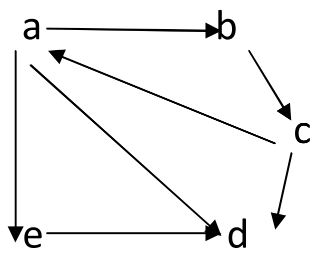
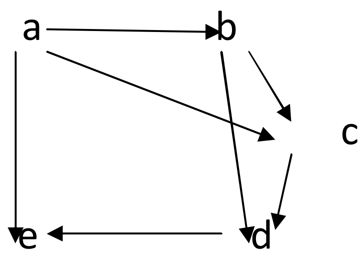
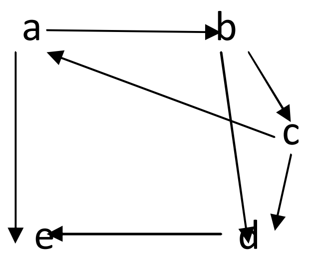
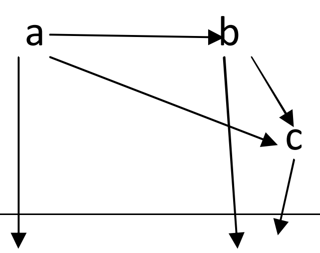
	In	out
a	2	2
b	2	0
c	2	0
d	0	2
e	0	2

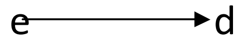
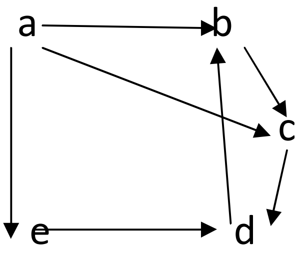
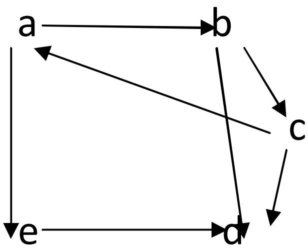
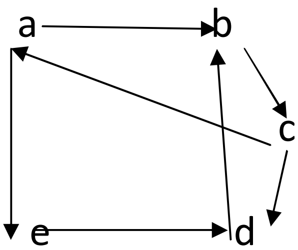
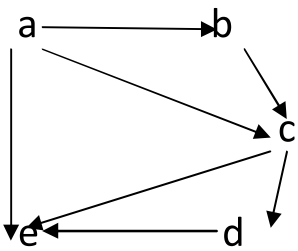
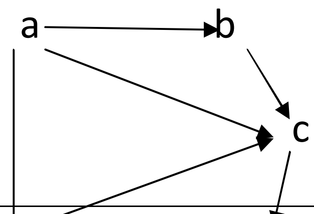
$G^{68}_{5,6}$		In out a 2 2 b 1 0 c 3 0 d 0 2 e 0 2
$G^{69}_{5,6}$		In out a 3 1 b 3 0 c 0 2 d 0 2 e 0 1
$G^{70}_{5,6}$		In out a 3 1 b 1 0 c 0 2 d 2 1 e 0 2
$G^{71}_{5,6}$		In out a 3 0 b 0 2 c 0 2 d 3 0 e 0 2
$G^{72}_{5,6}$		In out a 4 0 b 0 2 c 2 1 d 0 2 e 0 1

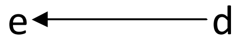
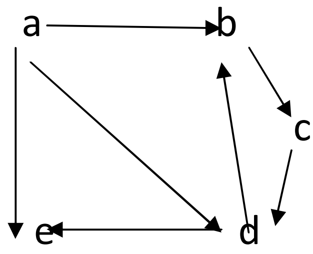
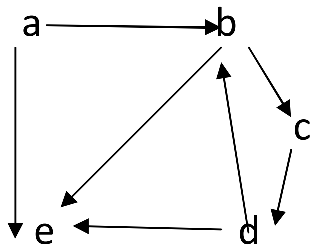
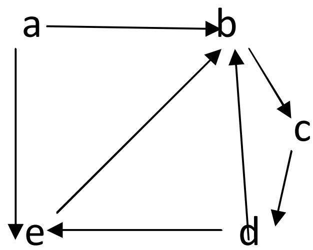
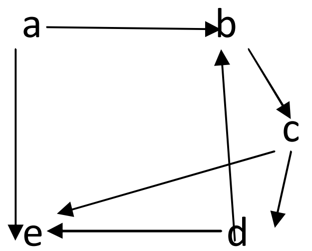
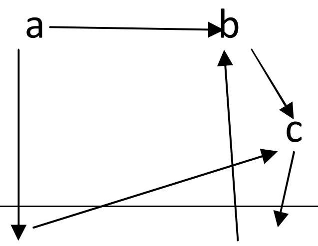
$G_{5,7}$(8

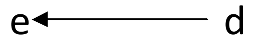
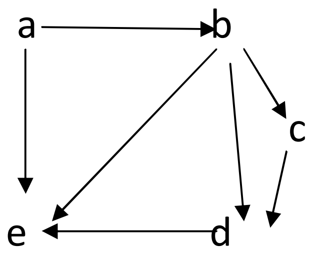
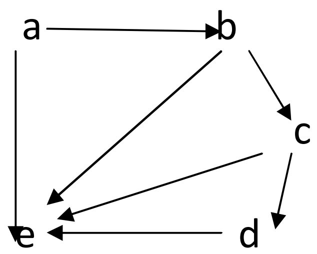
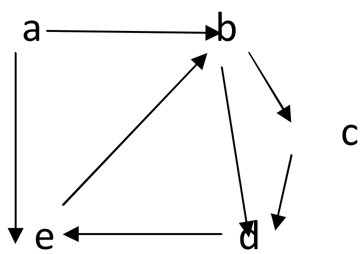
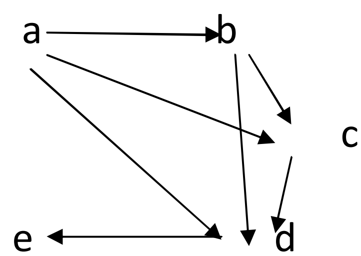
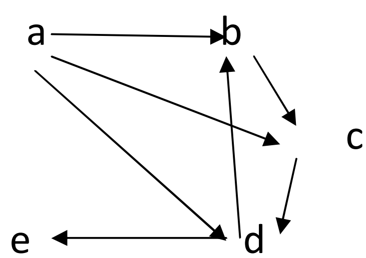
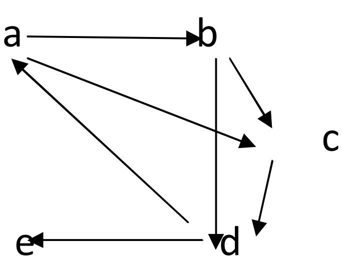
$G^{1}_{5,7}$		In out a 0 4 b 1 1

		c 2 1 d 2 1 e 2 0
$G^2_{5,7}$		In out a 1 3 b 1 1 c 2 1 d 2 1 e 1 1
$G^3_{5,7}$		In out a 2 2 b 1 1 c 2 1 d 1 2 e 1 1
$G^4_{5,7}$		In out a 3 1 b 1 1 c 1 2 d 1 2 e 1 1
$G^5_{5,7}$		In out a 1 3 b 1 1 c 2 1 d 1 2 e 2 0
$G^6_{5,7}$		In out a 2 2 b 1 1 c 1 2 d 1 2

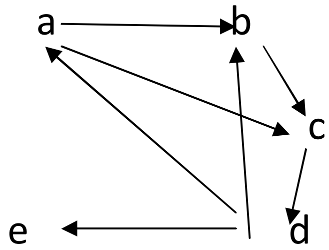
Graph	Diagram		In	out
		e	2	0
$G^{7}_{5,7}$			In	out
		a	0	4
		b	1	1
		c	2	1
		d	3	0
		e	1	1
$G^{8}_{5,7}$			In	out
		a	1	3
		b	1	1
		c	1	2
		d	3	0
		e	1	1
$G^{9}_{5,7}$			In	out
		a	0	3
		b	1	2
		c	2	1
		d	2	1
		e	2	0
$G^{10}_{5,7}$			In	out
		a	1	2
		b	1	2
		c	1	2
		d	2	1
		e	2	0
$G^{11}_{5,7}$			In	out
		a	0	3
		b	1	2
		c	2	1
		d	3	0

		**e 1 1**
$G^{12}_{5,7}$		In out a 0 3 b 2 1 c 2 1 d 2 1 e 1 1
$G^{13}_{5,7}$		In out a 1 2 b 1 2 c 1 2 d 3 0 e 1 1
$G^{14}_{5,7}$		In out a 1 2 b 2 1 c 1 2 d 2 1 e 1 1
$G^{15}_{5,7}$		In out a 0 3 b 1 1 c 2 2 d 1 1 e 3 0
$G^{16}_{5,7}$		In out a 0 3 b 1 1 c 3 1 d 1 1

		e 2 1
$G^{17}_{5,7}$		In out a 0 3 b 2 1 c 1 1 d 2 2 e 2 0
$G^{18}_{5,7}$		In out a 0 2 b 2 2 c 1 1 d 1 2 e 3 0
$G^{19}_{5,7}$		In out a 0 2 b 3 1 c 1 1 d 1 2 e 2 1
$G^{20}_{5,7}$		In out a 0 2 b 2 1 c 1 2 d 1 2 e 3 0
$G^{21}_{5,7}$		In out a 0 2 b 2 1 c 2 1 d 1 2

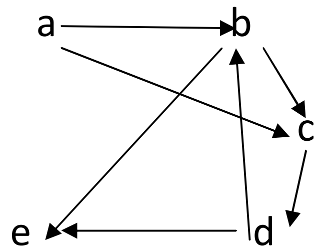
		e 2 1
$G^{22}_{5,7}$		In out a 0 2 b 1 3 c 1 1 d 2 1 e 3 0
$G^{23}_{5,7}$		In out a 0 2 b 1 2 c 1 2 d 1 1 e 4 0
$G^{24}_{5,7}$		In out a 0 2 b 2 2 c 1 1 d 2 1 e 2 1
$G^{25}_{5,7}$		In out a 0 3 b 1 2 c 2 1 d 3 1 e 1 0
$G^{26}_{5,7}$		In out a 0 3 b 2 1 c 2 1 d 2 2 e 1 0
$G^{27}_{5,7}$		In out a 1 2 b 1 2 c 2 1 d 2 2 e 1 0

$G^{28}_{5,7}$



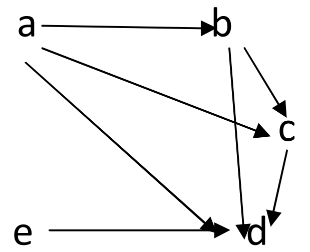
	In	out
a	1	2
b	2	1
c	2	1
d	1	3
e	1	0

$G^{29}_{5,7}$



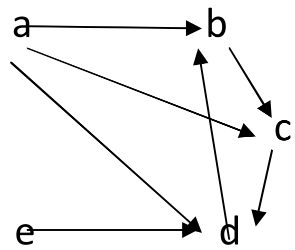
	In	out
a	0	2
b	2	2
c	2	1
d	1	2
e	2	0

$G^{30}_{5,7}$



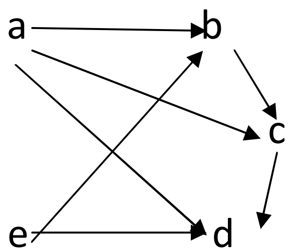
	In	out
a	0	3
b	1	2
c	2	1
d	4	0
e	0	1

$G^{31}_{5,7}$



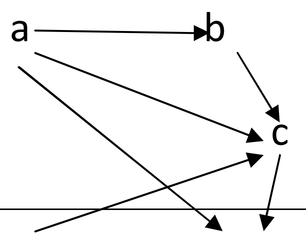
	In	out
a	0	3
b	2	1
c	2	1
d	3	1
e	0	1

$G^{32}_{5,7}$

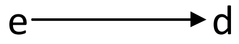
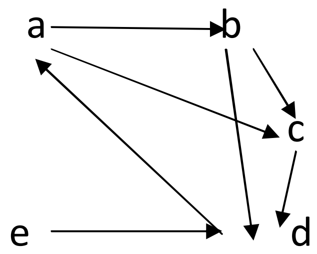
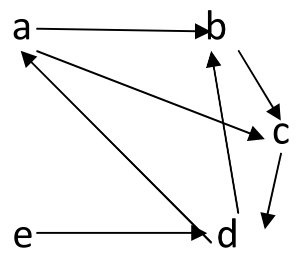
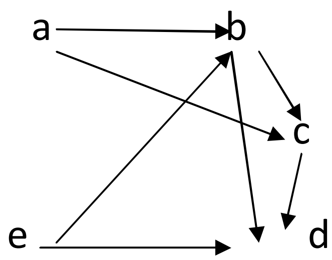
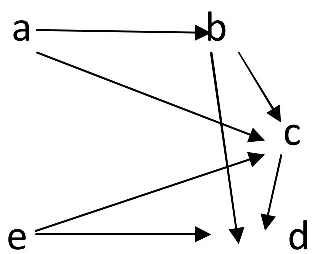
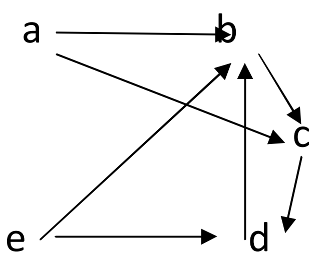


	In	out
a	0	3
b	2	1
c	2	1
d	3	0
e	0	2

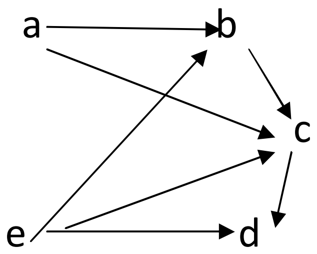
$G^{33}_{5,7}$



	In	out
a	0	3
b	1	1
c	3	1
d	3	0

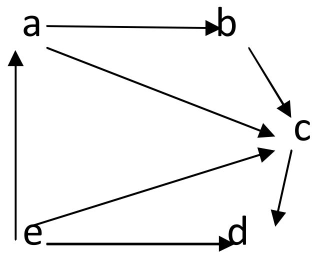
		e 0 2
$G^{34}_{5,7}$		In out a 1 2 b 1 2 c 2 1 d 3 1 e 0 1
$G^{35}_{5,7}$		In out a 1 2 b 2 1 c 2 1 d 2 2 e 0 1
$G^{36}_{5,7}$		In out a 0 2 b 2 2 c 2 1 d 3 0 e 0 2
$G^{37}_{5,7}$		In out a 0 2 b 1 2 c 3 1 d 3 0 e 0 2
$G^{38}_{5,7}$		In out a 0 2 b 3 1 c 2 1 d 2 1 e 0 2

$G^{39}_{5,7}$



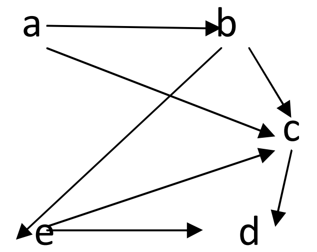
	In	out
a	0	2
b	2	1
c	3	1
d	2	0
e	0	3

$G^{40}_{5,7}$



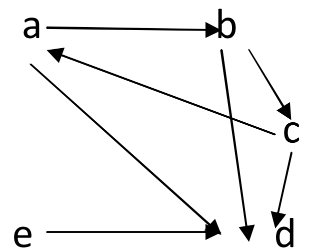
	In	out
a	1	2
b	1	1
c	3	1
d	2	0
e	0	3

$G^{41}_{5,7}$



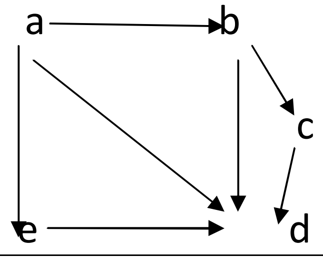
	In	out
a	0	2
b	1	2
c	3	1
d	2	0
e	1	2

$G^{42}_{5,7}$



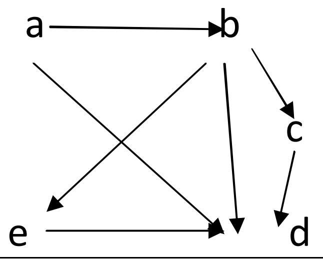
	In	out
a	1	2
b	1	2
c	1	2
d	4	0
e	0	1

$G^{43}_{5,7}$



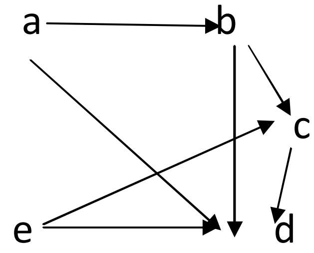
	In	out
a	0	3
b	1	2
c	1	1
d	4	0
e	1	1

$G^{44}_{5,7}$



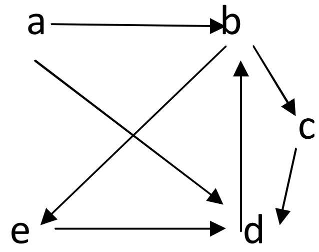
	In	out
a	0	2
b	1	3
c	1	1
d	4	0
e	1	1

$G^{45}_{5,7}$



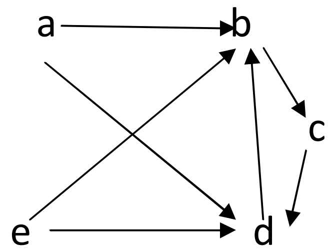
	In	out
a	0	2
b	1	2
c	2	1
d	4	0
e	0	2

$G^{46}_{5,7}$



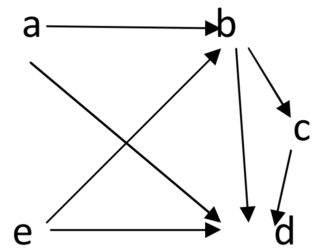
	In	out
a	0	2
b	2	2
c	1	1
d	3	1
e	1	1

$G^{47}_{5,7}$



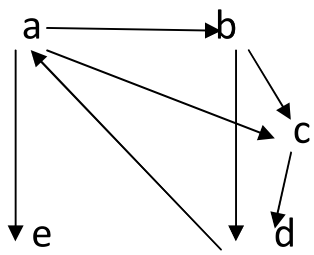
	In	out
a	0	2
b	3	1
c	1	1
d	3	1
e	0	2

$G^{48}_{5,7}$



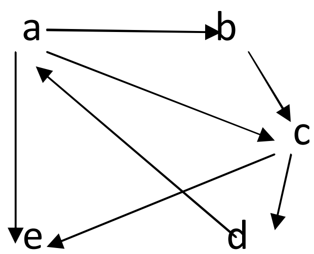
	In	out
a	0	2
b	2	2
c	1	1
d	4	0
e	0	2

$G^{49}_{5,7}$

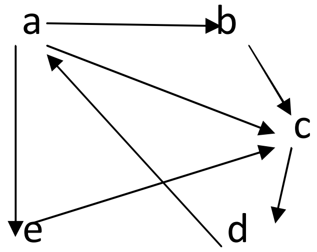
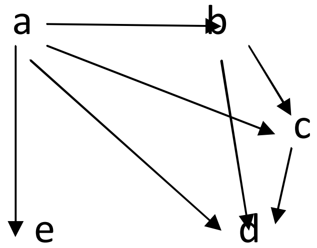
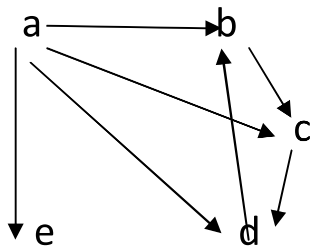
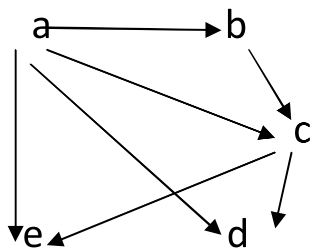
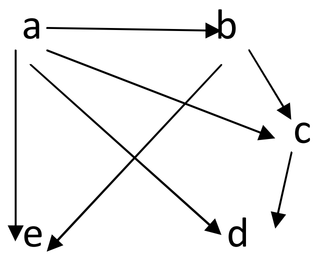
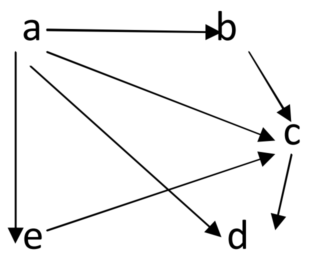


	In	out
a	1	3
b	1	2
c	2	1
d	2	1
e	1	0

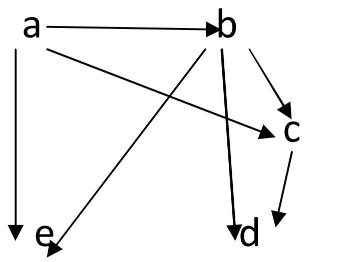
$G^{50}_{5,7}$



	In	out
a	1	3
b	1	1
c	2	2
d	1	1
e	2	0

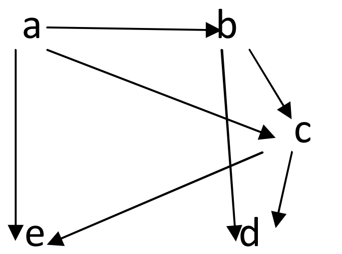
$G^{51}_{5,7}$		In out a 1 3 b 1 1 c 3 1 d 1 1 e 1 1
$G^{52}_{5,7}$		In out a 0 4 b 1 2 c 2 1 d 3 0 e 1 0
$G^{53}_{5,7}$		In out a 0 4 b 2 1 c 2 1 d 2 1 e 1 0
$G^{54}_{5,7}$		In out a 0 4 b 1 1 c 2 2 d 2 0 e 2 0
$G^{55}_{5,7}$		In out a 0 4 b 1 2 c 2 1 d 2 0 e 2 0
$G^{56}_{5,7}$		In out a 0 4 b 1 1 c 3 1 d 2 0 e 1 1

$G^{57}_{5,7}$



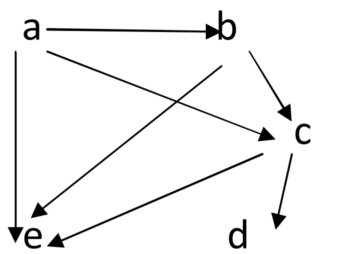
	In	out
a	0	3
b	1	3
c	2	1
d	2	0
e	2	0

$G^{58}_{5,7}$



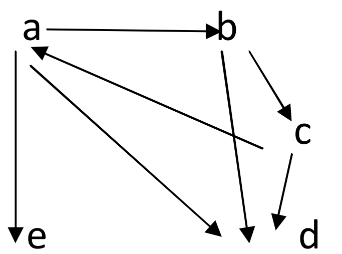
	In	out
a	0	3
b	1	2
c	2	2
d	2	0
e	2	0

$G^{59}_{5,7}$



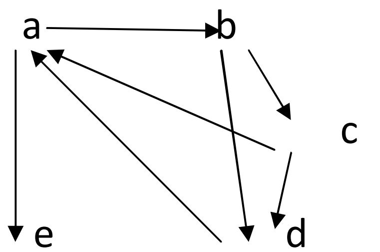
	In	out
a	0	3
b	1	2
c	2	2
d	1	0
e	3	0

$G^{60}_{5,7}$



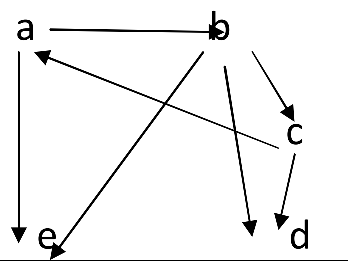
	In	out
a	1	3
b	1	2
c	1	2
d	3	0
e	1	0

$G^{61}_{5,7}$

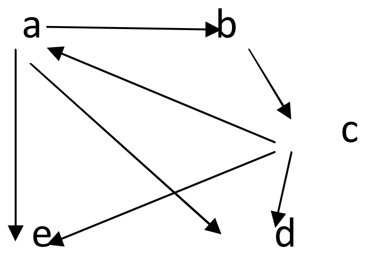
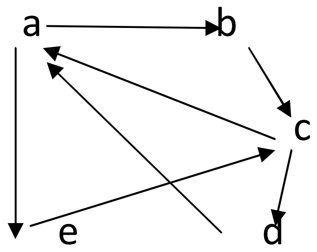
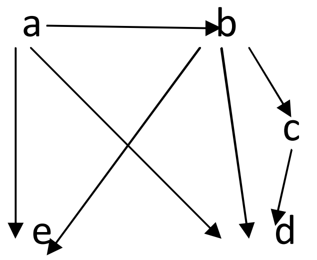
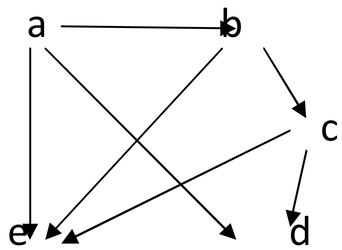
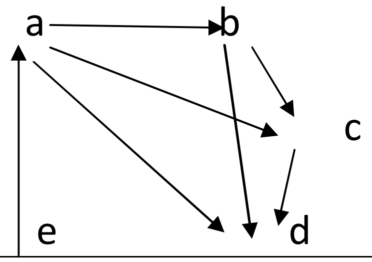
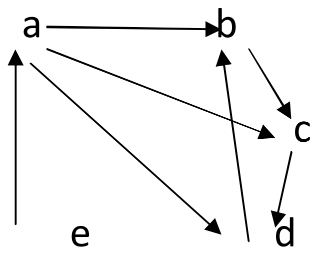


	In	out
a	2	2
b	1	2
c	1	2
d	2	1
e	1	0

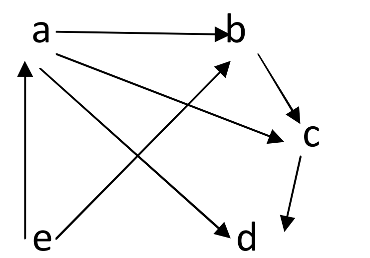
$G^{62}_{5,7}$



	In	out
a	1	2
b	1	3
c	1	2
d	2	0
e	2	0

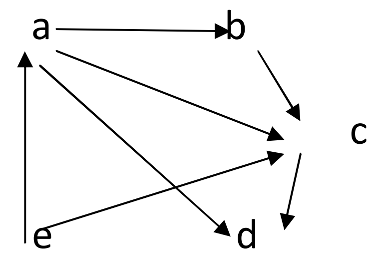
Graph	Diagram	Degrees
$G^{63}_{5,7}$		In out a 1 3 b 1 1 c 1 3 d 2 0 e 2 0
$G^{64}_{5,7}$		In out a 2 2 b 1 1 c 2 2 d 1 1 e 1 1
$G^{65}_{5,7}$		In out a 0 3 b 1 3 c 1 1 d 3 0 e 2 0
$G^{66}_{5,7}$		In out a 0 3 b 1 2 c 1 2 d 2 0 e 3 0
$G^{67}_{5,7}$		In out a 1 3 b 1 2 c 2 1 d 3 0 e 0 1
$G^{68}_{5,7}$		In out a 1 3 b 2 1 c 2 1 d 2 1 e 0 1

$G^{69}_{5,7}$



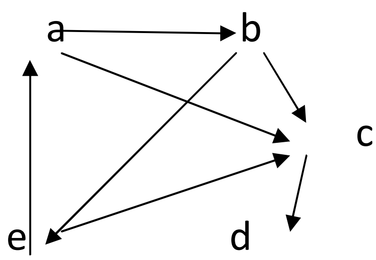
	In	out
a	1	3
b	2	1
c	2	1
d	2	0
e	0	2

$G^{70}_{5,7}$



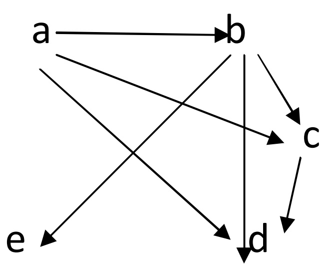
	In	out
a	1	3
b	1	1
c	3	1
d	2	0
e	0	2

$G^{71}_{5,7}$



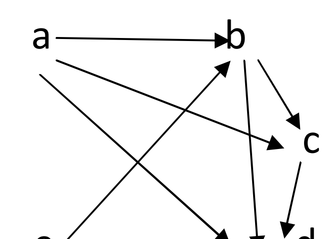
	In	out
a	1	2
b	1	2
c	3	1
d	1	0
e	1	2

$G^{72}_{5,7}$



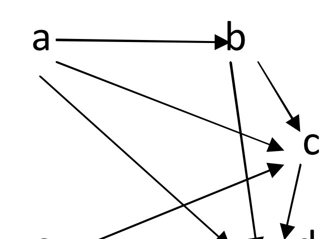
	In	out
a	0	3
b	1	3
c	2	1
d	3	0
e	1	0

$G^{73}_{5,7}$

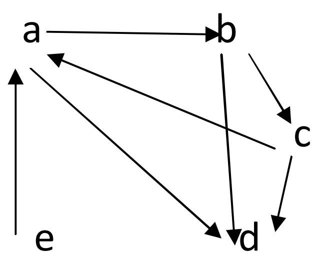
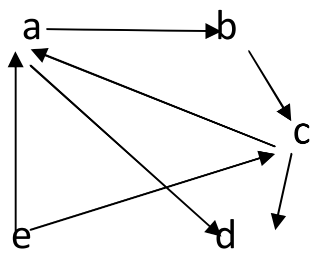
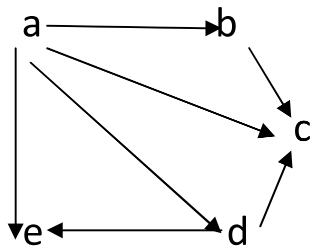
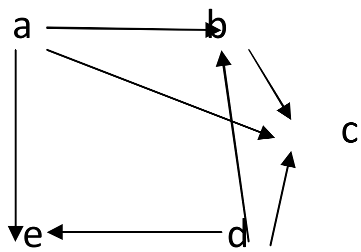
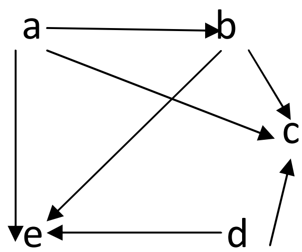
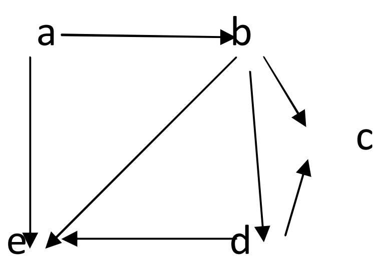


	In	out
a	0	3
b	2	2
c	2	1
d	3	0
e	0	1

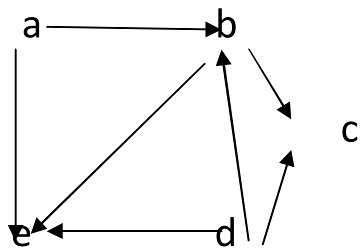
$G^{74}_{5,7}$



	In	out
a	0	3
b	1	2
c	3	1
d	3	0
e	0	1

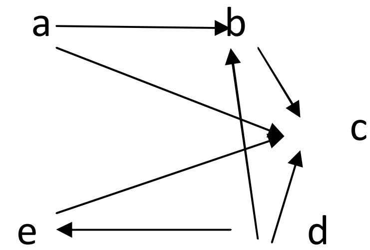
$G^{75}_{5,7}$		In out a 2 2 b 1 2 c 1 2 d 3 0 e 0 1
$G^{76}_{5,7}$		In out a 2 2 b 1 1 c 2 2 d 2 0 e 0 2
$G^{77}_{5,7}$		In out a 0 4 b 1 1 c 3 0 d 1 2 e 2 0
$G^{78}_{5,7}$		In out a 0 3 b 2 1 c 3 0 d 0 3 e 2 0
$G^{79}_{5,7}$		In out a 0 3 b 1 2 c 3 0 d 0 2 e 3 0
$G^{80}_{5,7}$		In out a 0 2 b 1 3 c 2 0 d 1 2 e 3 0

$G^{81}_{5,7}$



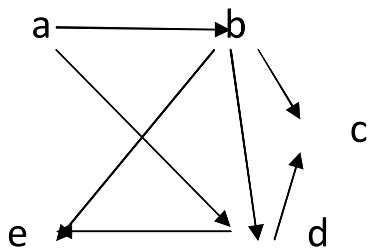
	In	out
a	0	2
b	2	2
c	2	0
d	0	3
e	3	0

$G^{82}_{5,7}$



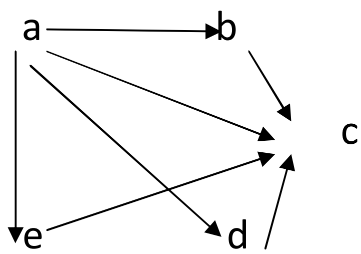
	In	out
a	0	2
b	2	1
c	4	0
d	0	3
e	1	1

$G^{83}_{5,7}$



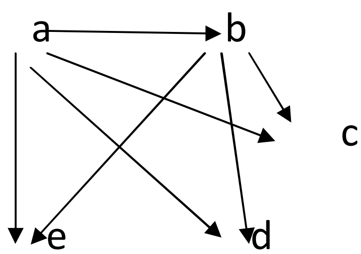
	In	out
a	0	2
b	1	3
c	2	0
d	2	2
e	2	0

$G^{84}_{5,7}$



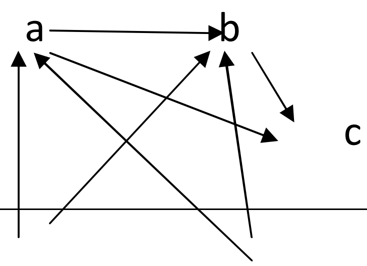
	In	out
a	0	4
b	1	1
c	4	0
d	1	1
e	1	1

$G^{85}_{5,7}$

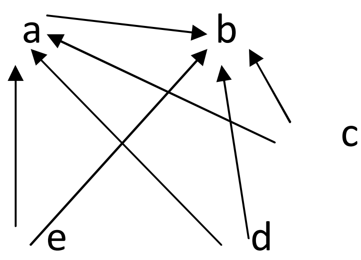


	In	out
a	0	4
b	1	3
c	2	0
d	2	0
e	2	0

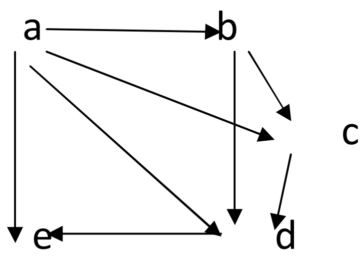
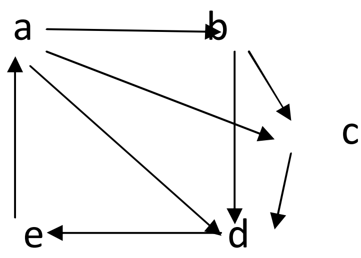
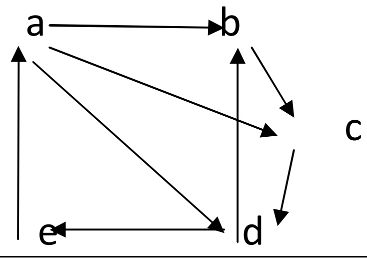
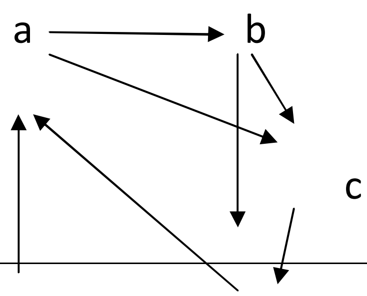
$G^{86}_{5,7}$

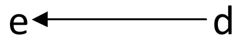
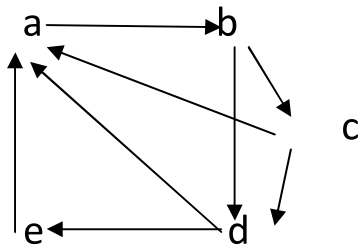
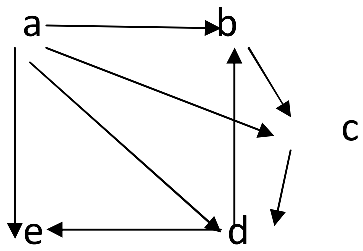
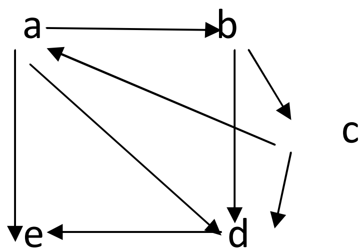
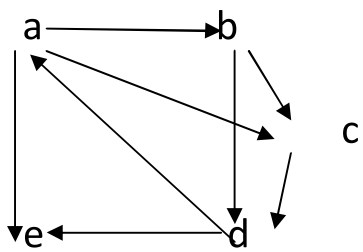
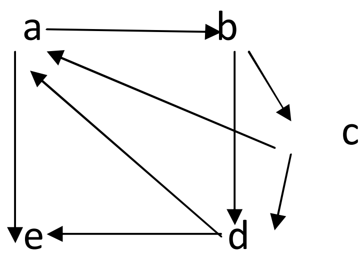
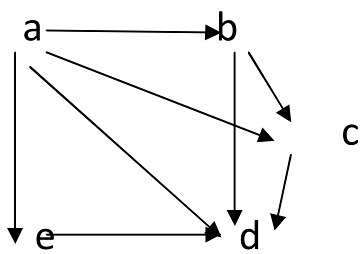


	In	out
a	2	2
b	3	1
c	2	0
d	0	2

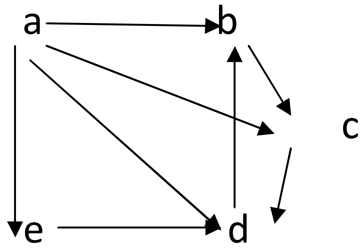
	e d	e 0 2
$G^{87}_{5,7}$		In out a 3 1 b 4 0 c 0 2 d 0 2 e 0 2

$G_{5,8}$ (9

$G^{1}_{5,8}$		In out a 0 4 b 1 2 c 2 1 d 2 1 e 2 0
$G^{2}_{5,8}$		In out a 1 3 b 1 2 c 2 1 d 3 1 e 1 1
$G^{3}_{5,8}$		In out a 1 3 b 2 1 c 2 1 d 2 2 e 1 1
$G^{4}_{5,8}$		In out a 2 2 b 1 2 c 2 1 d 2 2

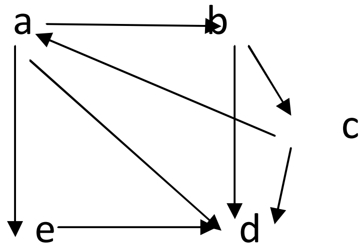
		e 1 1
$G^5_{5,8}$		In out a 3 1 b 1 2 c 1 2 d 2 2 e 1 1
$G^6_{5,8}$		In out a 0 4 b 2 1 c 2 1 d 2 2 e 2 0
$G^7_{5,8}$		In out a 1 3 b 1 2 c 1 2 d 3 1 e 2 0
$G^8_{5,8}$		In out a 1 3 b 1 2 c 2 1 d 2 2 e 2 0
$G^9_{5,8}$		In out a 2 2 b 1 2 c 1 2 d 2 2 e 2 0
$G^{10}_{5,8}$		In out a 0 4 b 1 2 c 2 1 d 4 0 e 1 1

$G^{11}_{5,8}$



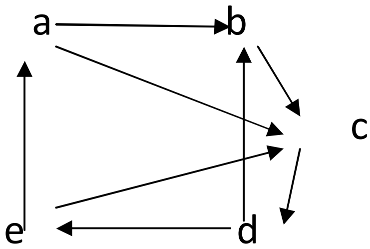
	In	out
a	0	4
b	2	1
c	2	1
d	3	1
e	1	1

$G^{12}_{5,8}$



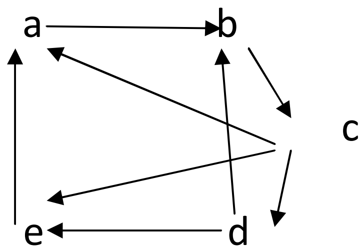
	In	out
a	1	3
b	1	2
c	1	2
d	4	0
e	1	1

$G^{13}_{5,8}$



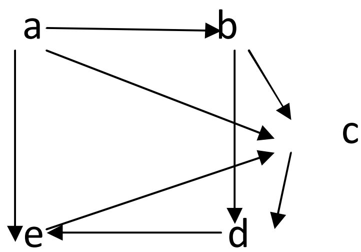
	In	out
a	1	2
b	2	1
c	3	1
d	1	2
e	1	2

$G^{14}_{5,8}$



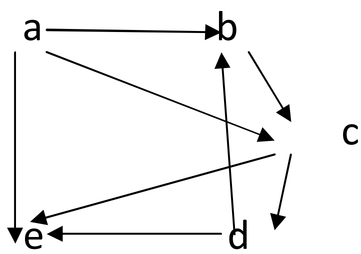
	In	out
a	2	1
b	2	1
c	1	3
d	1	2
e	2	1

$G^{15}_{5,8}$



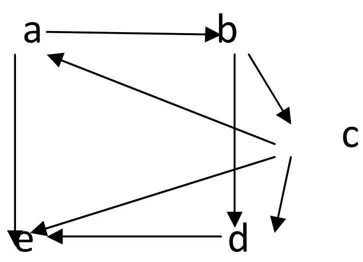
	In	out
a	0	3
b	1	2
c	3	1
d	2	1
e	2	1

$G^{16}_{5,8}$



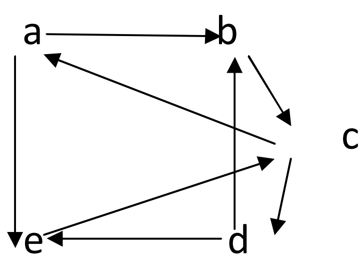
	In	out
a	0	3
b	2	1
c	2	2
d	1	2
e	3	0

$G^{17}_{5,8}$



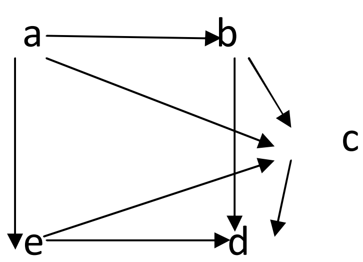
	In	out
a	1	2
b	1	2
c	1	3
d	2	1
e	3	0

$G^{18}_{5,8}$



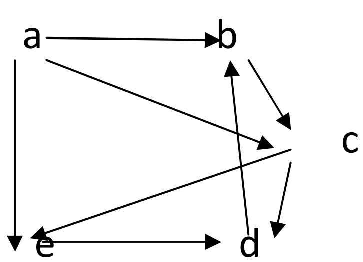
	In	out
a	1	2
b	2	1
c	2	2
d	1	2
e	2	1

$G^{19}_{5,8}$



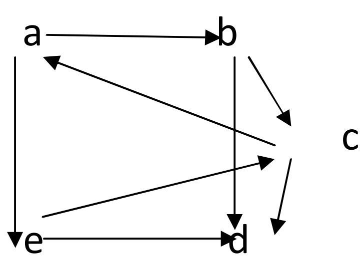
	In	out
a	0	3
b	1	2
c	3	1
d	3	0
e	1	2

$G^{20}_{5,8}$



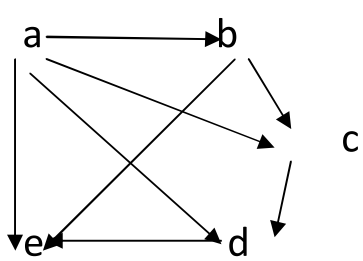
	In	out
a	0	3
b	2	1
c	2	2
d	2	1
e	2	1

$G^{21}_{5,8}$

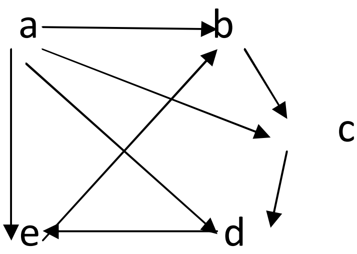
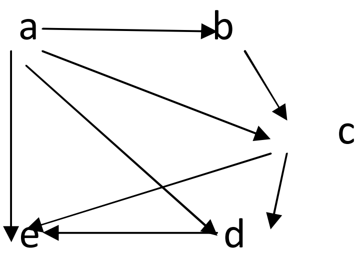
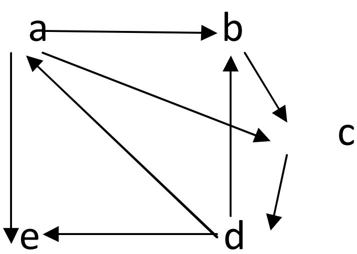
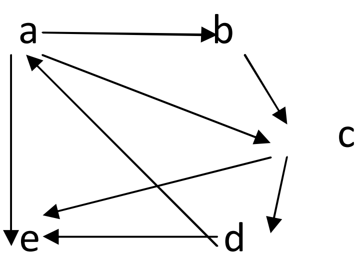
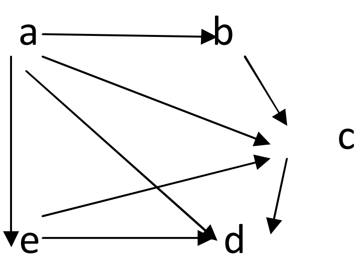
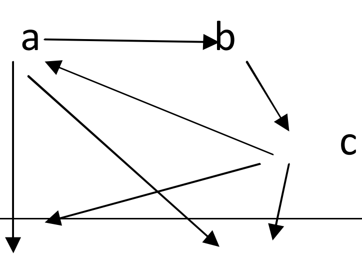


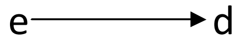
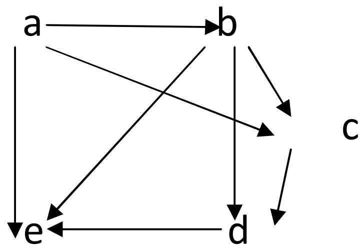
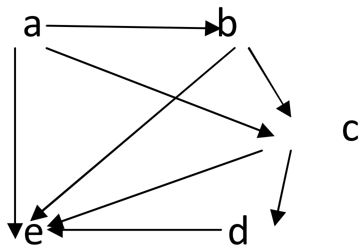
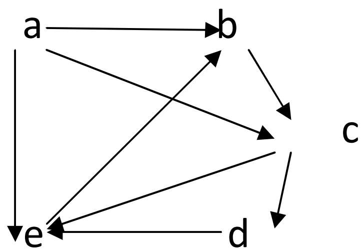
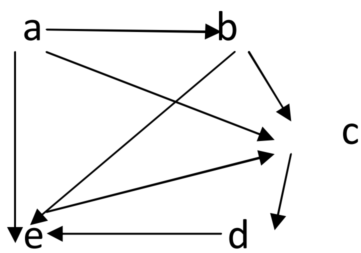
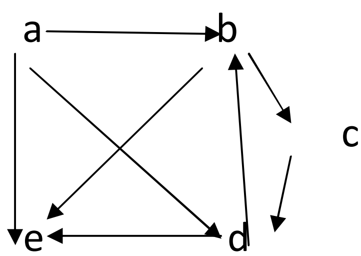
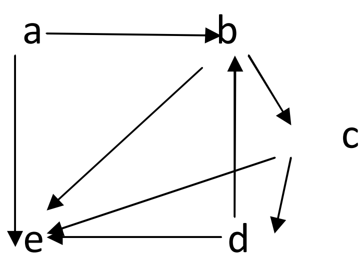
	In	out
a	1	2
b	1	2
c	2	2
d	3	0
e	1	2

$G^{22}_{5,8}$

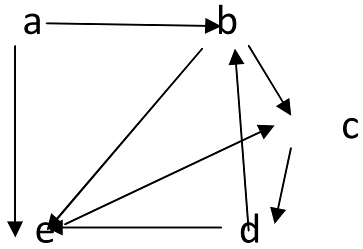


	In	out
a	0	4
b	1	2
c	2	1
d	2	1
e	3	0

$G^{23}_{5,8}$	 In out a 0 4 b 2 1 c 2 1 d 2 1 e 2 1
$G^{24}_{5,8}$	 In out a 0 4 b 1 1 c 2 2 d 2 1 e 3 0
$G^{25}_{5,8}$	 In out a 1 3 b 2 1 c 2 1 d 1 3 e 2 0
$G^{26}_{5,8}$	 In out a 1 3 b 1 1 c 2 2 d 1 2 e 3 0
$G^{27}_{5,8}$	 In out a 0 4 b 1 1 c 3 1 d 3 0 e 1 2
$G^{28}_{5,8}$	 In out a 1 3 b 1 1 c 1 3 d 3 0

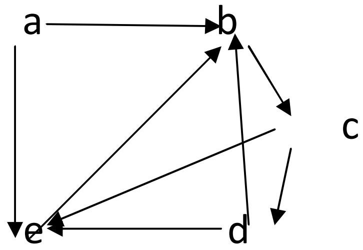
		e 2 1
$G^{29}_{5,8}$		In out a 0 3 b 1 3 c 2 1 d 2 1 e 3 0
$G^{30}_{5,8}$		In out a 0 3 b 1 2 c 2 2 d 1 1 e 4 0
$G^{31}_{5,8}$		In out a 0 3 b 2 1 c 2 2 d 1 1 e 3 1
$G^{32}_{5,8}$		In out a 0 3 b 1 2 c 3 1 d 1 1 e 3 1
$G^{33}_{5,8}$		In out a 0 3 b 2 2 c 1 1 d 2 2 e 3 0
$G^{34}_{5,8}$		In out a 0 2 b 2 2 c 1 2 d 1 2 e 4 0

$G^{35}_{5,8}$



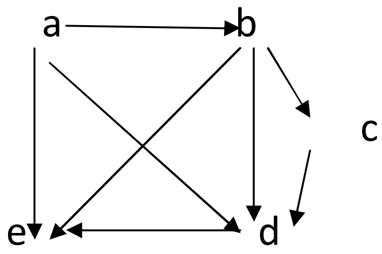
	In	out
a	0	2
b	2	2
c	2	1
d	1	2
e	3	1

$G^{36}_{5,8}$



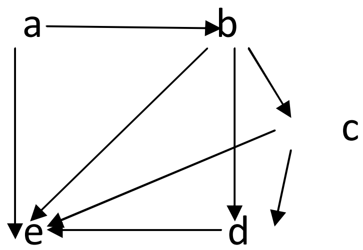
	In	out
a	0	2
b	3	1
c	1	2
d	1	2
e	3	1

$G^{37}_{5,8}$



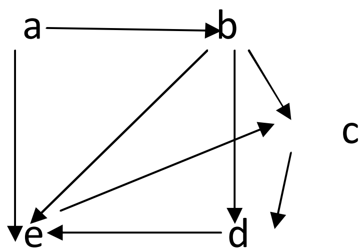
	In	out
a	0	3
b	1	3
c	1	1
d	3	1
e	3	0

$G^{38}_{5,8}$



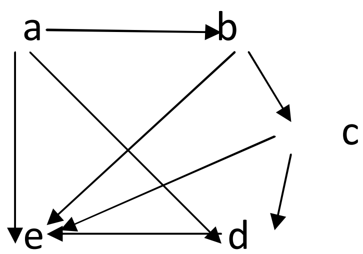
	In	out
a	0	2
b	1	3
c	1	2
d	2	1
e	4	0

$G^{39}_{5,8}$



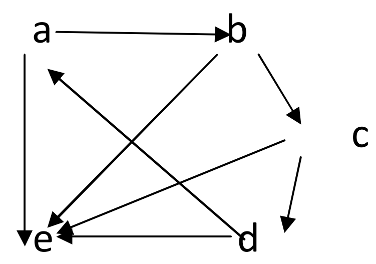
	In	out
a	0	2
b	1	3
c	2	1
d	2	1
e	3	1

$G^{40}_{5,8}$



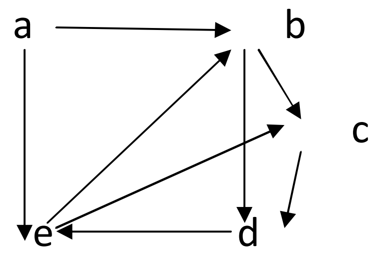
	In	out
a	0	3
b	1	2
c	1	2
d	2	1
e	4	0

$G^{41}_{5,8}$



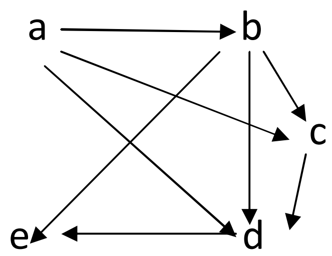
	In	out
a	1	2
b	1	2
c	1	2
d	1	2
e	4	0

$G^{42}_{5,8}$



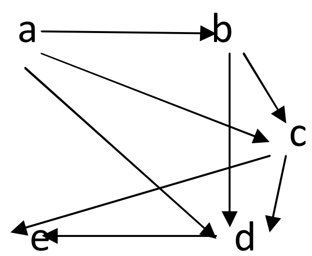
	In	out
a	0	2
b	2	2
c	2	1
d	2	1
e	2	2

$G^{43}_{5,8}$



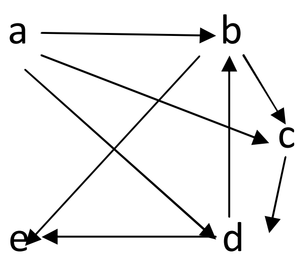
	In	out
a	0	3
b	1	3
c	2	1
d	3	1
e	2	0

$G^{44}_{5,8}$



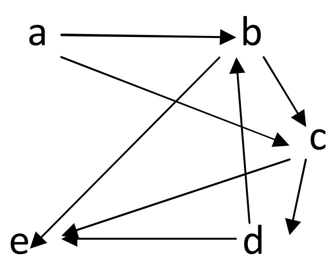
	In	out
a	0	3
b	1	2
c	2	2
d	3	1
e	2	0

$G^{45}_{5,8}$



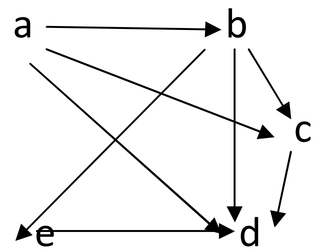
	In	out
a	0	3
b	2	2
c	2	1
d	2	2
e	2	0

$G^{46}_{5,8}$



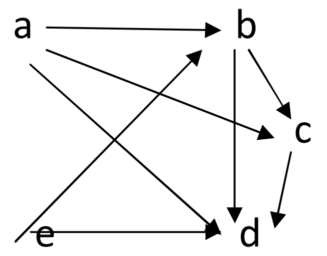
	In	out
a	0	2
b	2	2
c	2	2
d	1	2
e	3	0

$G^{47}_{5,8}$



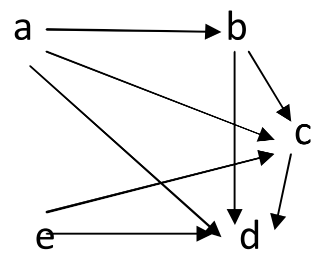
	In	out
a	0	3
b	1	3
c	2	1
d	4	0
e	1	1

$G^{48}_{5,8}$



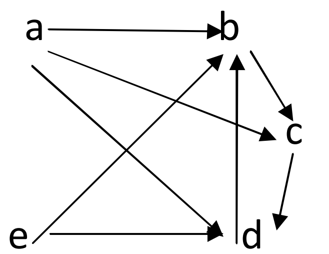
	In	out
a	0	3
b	2	2
c	2	1
d	4	0
e	0	2

$G^{49}_{5,8}$



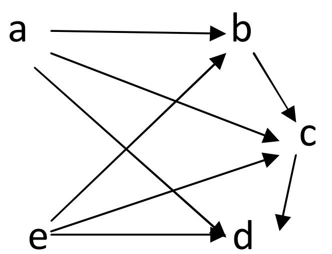
	In	out
a	0	3
b	1	2
c	3	1
d	4	0
e	0	2

$G^{50}_{5,8}$



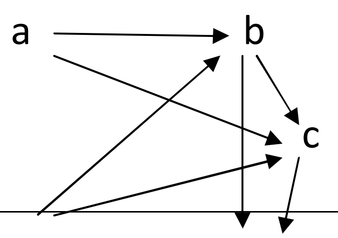
	In	out
a	0	3
b	3	1
c	2	1
d	3	1
e	0	2

$G^{51}_{5,8}$

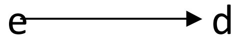
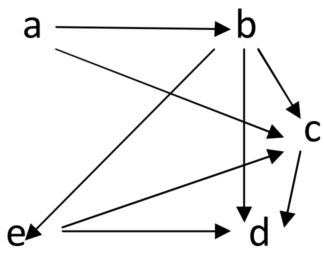
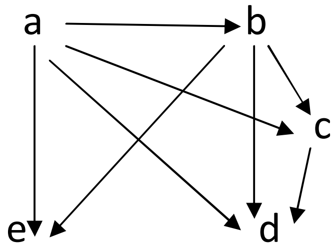
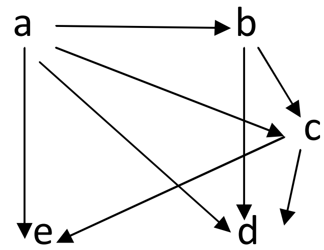
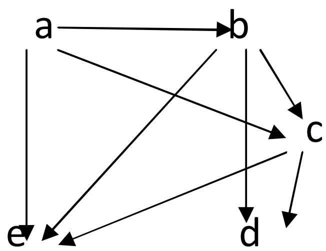
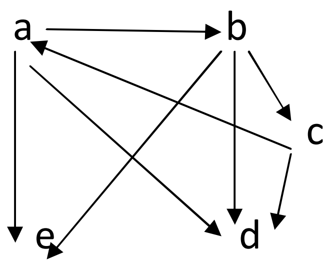
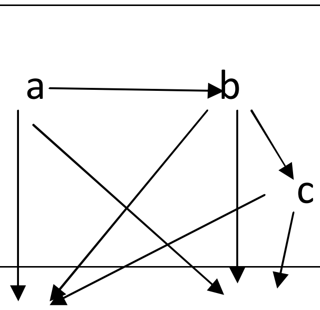


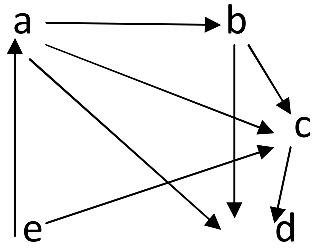
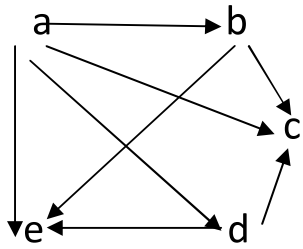
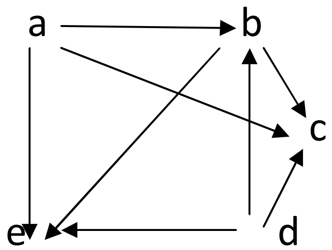
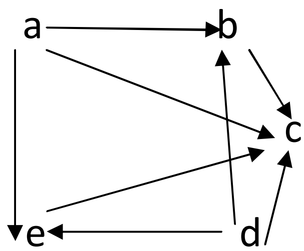
	In	out
a	0	3
b	2	1
c	3	1
d	3	0
e	0	3

$G^{52}_{5,8}$

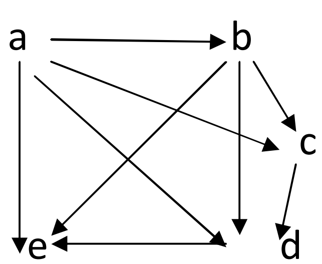
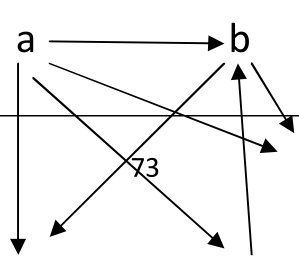


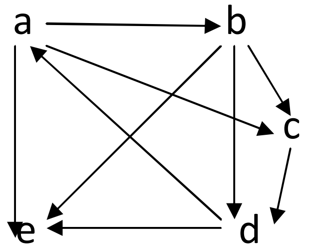
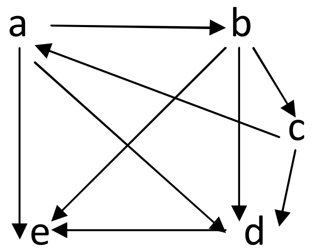
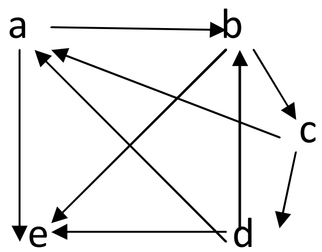
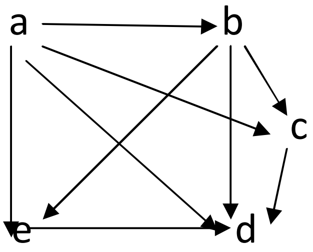
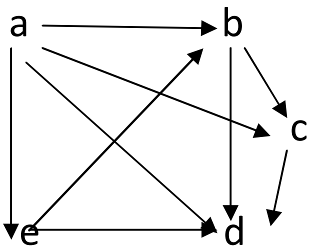
	In	out
a	0	2
b	2	2
c	3	1
d	3	0

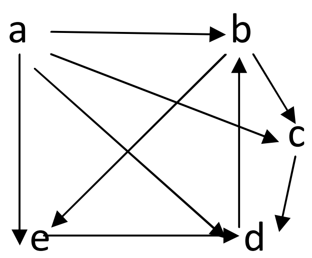
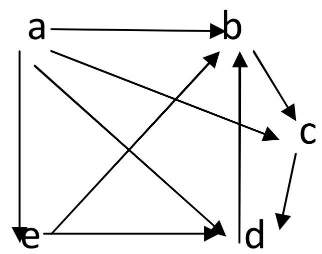
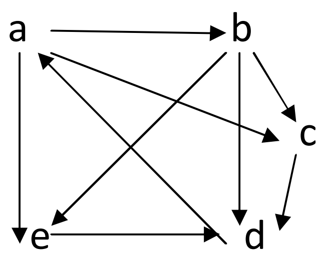
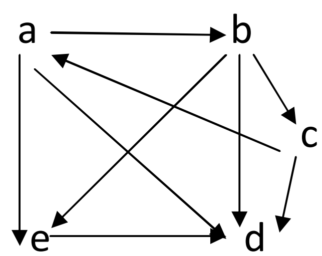
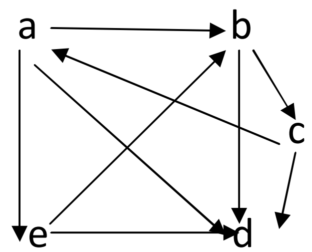
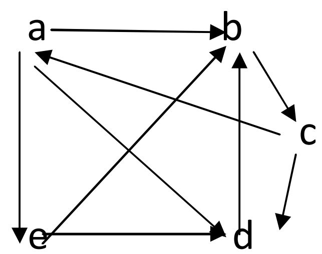
		e 0 3
$G^{53}_{5,8}$		In out a 0 2 b 1 3 c 3 1 d 3 0 e 1 2
$G^{54}_{5,8}$		In out a 0 4 b 1 3 c 2 1 d 3 0 e 2 0
$G^{55}_{5,8}$		In out a 0 4 b 1 2 c 2 2 d 3 0 e 0 2
$G^{56}_{5,8}$		In out a 0 3 b 1 3 c 2 2 d 2 0 e 3 0
$G^{57}_{5,8}$		In out a 1 3 b 1 3 c 1 2 d 3 0 e 2 0
$G^{58}_{5,8}$		In out a 0 3 b 1 3 c 1 2 d 3 0

	e d	e 3 0
$G^{59}_{5,8}$		In out a 1 3 b 1 2 c 3 1 d 3 0 e 0 2
$G^{60}_{5,8}$		In out a 0 4 b 1 2 c 3 0 d 1 2 e 3 0
$G^{61}_{5,8}$		In out a 0 3 b 2 2 c 3 0 d 0 3 e 3 0
$G^{62}_{5,8}$		In out a 0 3 b 2 1 c 4 0 d 0 3 e 2 1

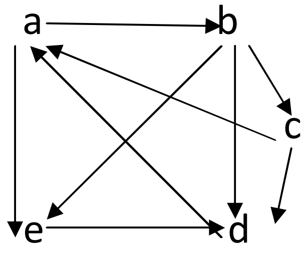
$G_{5,9}$ (10

$G^{1}_{5,9}$		In out a 0 4 b 1 3 c 2 1 d 3 1 e 3 0
$G^{2}_{5,9}$		In out a 0 4 b 2 2

		c 2 1 d 2 2 e 3 0
$G^3_{5,9}$		In out a 1 3 b 1 3 c 2 1 d 2 2 e 3 0
$G^4_{5,9}$		In out a 1 3 b 1 3 c 1 2 d 3 1 e 3 0
$G^5_{5,9}$		In out a 2 2 b 2 2 c 1 2 d 1 3 e 3 0
$G^6_{5,9}$		In out a 0 4 b 1 3 c 2 1 d 4 0 e 2 1
$G^7_{5,9}$		In out a 0 4 b 2 2 c 2 1 d 4 0 e 1 2

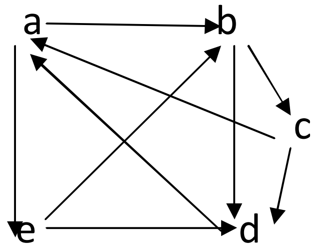
$G^{8}_{5,9}$		In out a 0 4 b 2 2 c 2 1 d 3 1 e 2 1
$G^{9}_{5,9}$		In out a 0 4 b 3 1 c 2 1 d 3 1 e 1 2
$G^{10}_{5,9}$		In out a 1 3 b 1 3 c 2 1 d 3 1 e 2 1
$G^{11}_{5,9}$		In out a 1 3 b 1 3 c 1 2 d 4 0 e 2 1
$G^{12}_{5,9}$		In out a 1 3 b 2 2 c 1 2 d 4 0 e 1 2
$G^{13}_{5,9}$		In out a 1 3 b 3 1 c 1 2 d 3 1 e 1 2

$G^{14}_{5,9}$



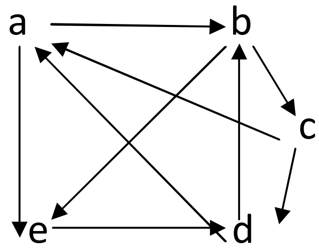
	In	out
a	2	2
b	1	3
c	1	2
d	3	1
e	2	1

$G^{15}_{5,9}$



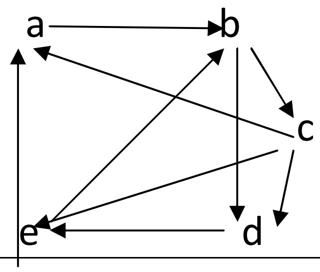
	In	out
a	2	2
b	2	2
c	1	2
d	3	1
e	1	2

$G^{16}_{5,9}$



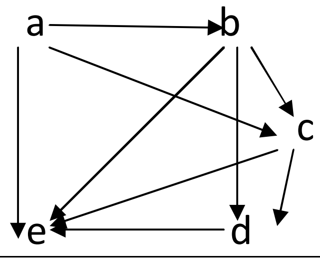
	In	out
a	2	2
b	2	2
c	1	2
d	2	2
e	2	1

$G^{17}_{5,9}$



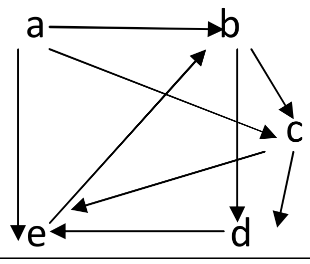
	In	out
a	2	1
b	2	2
c	1	3
d	2	1
e	2	2

$G^{18}_{5,9}$



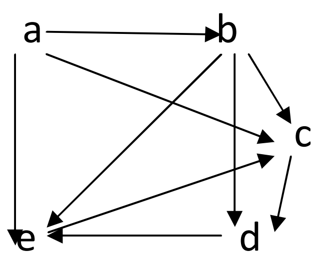
	In	out
a	0	3
b	1	3
c	2	2
d	2	1
e	4	0

$G^{19}_{5,9}$



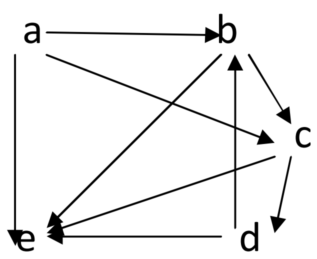
	In	out
a	0	3
b	2	2
c	2	2
d	2	1
e	3	1

$G^{20}_{5,9}$



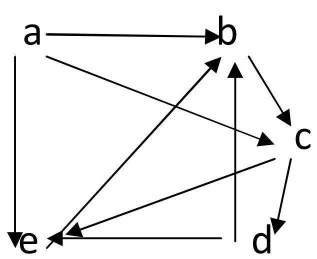
	In	out
a	0	3
b	1	3
c	3	1
d	2	1
e	3	1

$G^{21}_{5,9}$



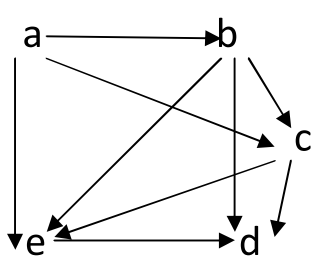
	In	out
a	0	3
b	2	2
c	2	2
d	1	2
e	4	0

$G^{22}_{5,9}$



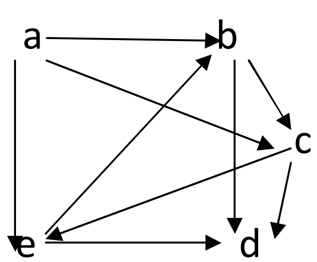
	In	out
a	0	3
b	3	1
c	2	2
d	1	2
e	3	1

$G^{23}_{5,9}$



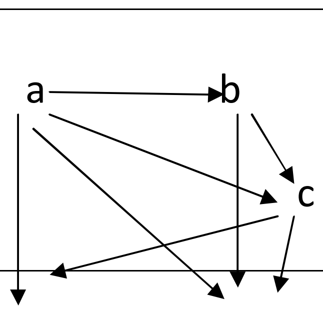
	In	out
a	0	3
b	1	3
c	2	2
d	3	0
e	3	1

$G^{24}_{5,9}$

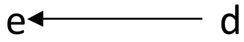
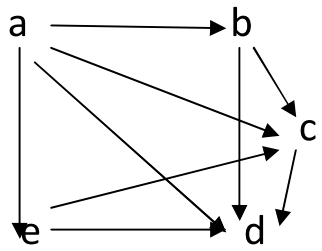
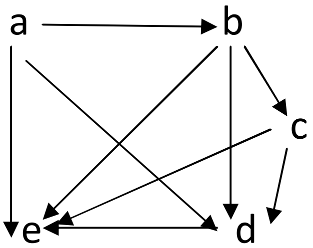
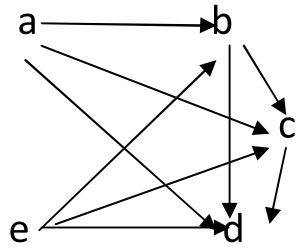
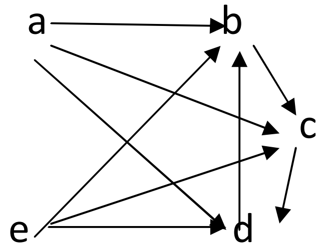
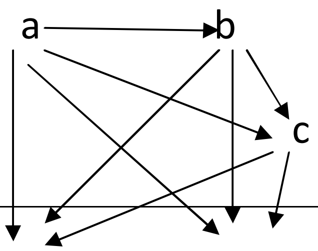


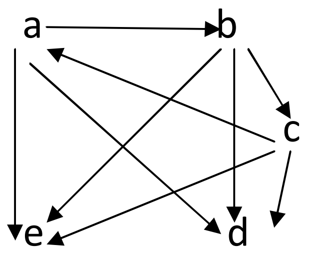
	In	out
a	0	3
b	2	2
c	2	2
d	3	0
e	2	2

$G^{25}_{5,9}$

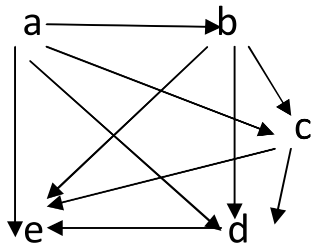
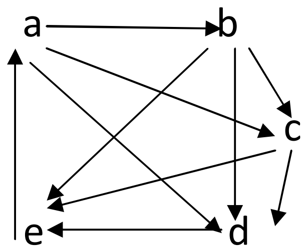
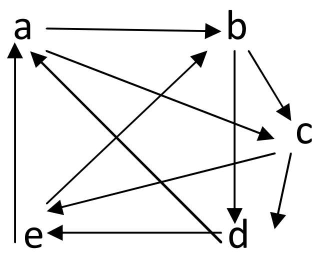
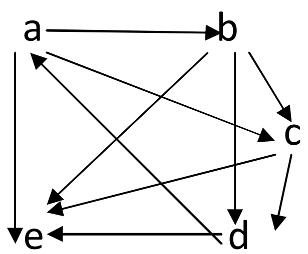


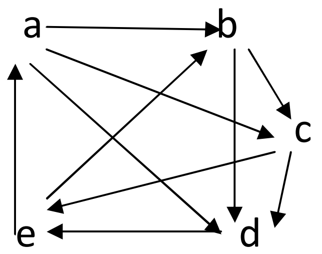
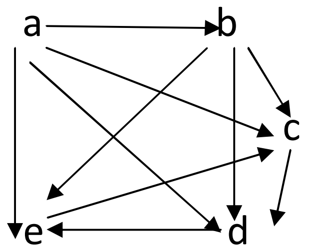
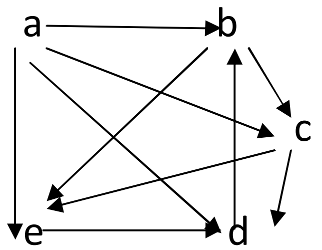
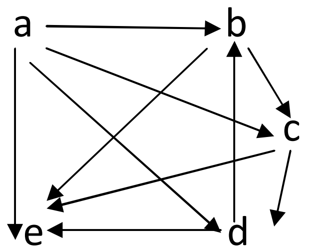
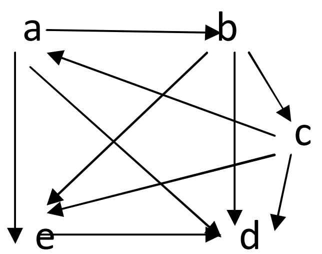
	In	out
a	0	4
b	1	2
c	2	2
d	3	1

		e 3 0
$G^{26}_{5,9}$		In out a 0 4 b 1 2 c 3 1 d 4 0 e 1 2
$G^{27}_{5,9}$		In out a 0 3 b 1 3 c 1 2 d 3 1 e 4 0
$G^{28}_{5,9}$		In out a 0 3 b 2 2 c 3 1 d 4 0 e 0 3
$G^{29}_{5,9}$		In out a 0 3 b 3 1 c 3 1 d 3 1 e 0 3
$G^{30}_{5,9}$		In out a 0 4 b 1 3 c 2 2 d 3 0

	e d	e 3 0
$G^{31}_{5,9}$		In out a 1 3 b 1 3 c 1 3 d 3 0 e 3 0

$G_{5,10}$(11

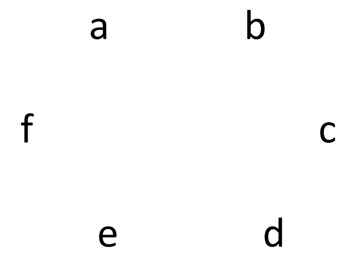
$G^{1}_{5,10}$		In out a 0 4 b 1 3 c 2 2 d 3 1 e 4 0
$G^{2}_{5,10}$		In out a 1 3 b 1 3 c 2 2 d 3 1 e 3 1
$G^{3}_{5,10}$		In out a 2 2 b 2 2 c 2 2 d 2 2 e 2 2
$G^{4}_{5,10}$		In out a 1 3 b 1 3 c 2 2 d 2 2 e 4 0

$G^5_{5,10}$		In out a 1 3 b 2 2 c 2 2 d 3 1 e 2 2
$G^6_{5,10}$		In out a 0 4 b 1 3 c 3 1 d 3 1 e 3 1
$G^7_{5,10}$		In out a 0 4 b 2 2 c 2 2 d 3 1 e 3 1
$G^8_{5,10}$		In out a 0 4 b 2 2 c 2 2 d 2 2 e 4 0
$G^9_{5,10}$		In out a 1 3 b 1 3 c 1 3 d 4 0 e 3 1

(2-4-7) البيانات المولدة بالمجموعة 6 (G^6) :

1) $G_{6,0}$:

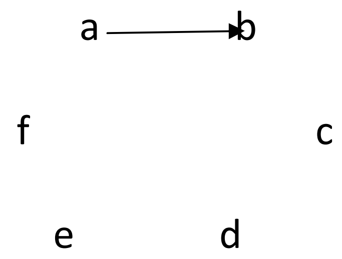
$G^1_{6,0}$	In out



	In	out
a	0	0
b	0	0
c	0	0
d	0	0
e	0	0
f	0	0

: $G_{6,1}$ (2

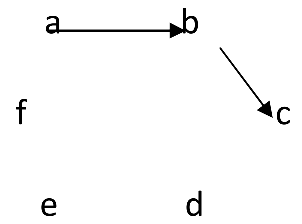
$G^{1}_{6,1}$



	In	out
a	0	1
b	1	0
c	0	0
d	0	0
e	0	0
f	0	0

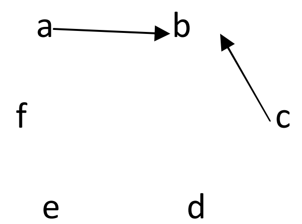
: $G_{6,2}$ (3

$G^{1}_{6,2}$



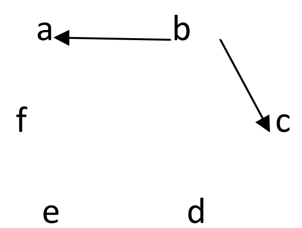
	In	out
a	0	1
b	1	1
c	1	0
d	0	0
e	0	0
f	0	0

$G^{2}_{6,2}$



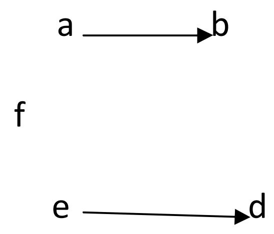
	In	out
a	0	1
b	2	0
c	0	1
d	0	0
e	0	0
f	0	0

$G^{3}_{6,2}$



	In	out
a	1	0
b	0	2
c	1	0
d	0	0
e	0	0
f	0	0

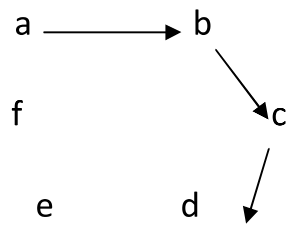
$G^{4}_{6,2}$



	In	out
a	0	1
b	1	0
c	0	0
d	1	0
e	0	1
f	0	0

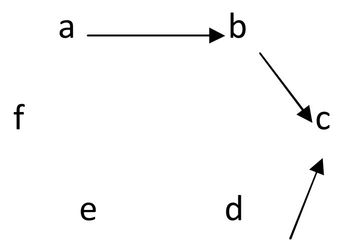
: $G_{6,3}$ (4

$G^{1}_{6,3}$



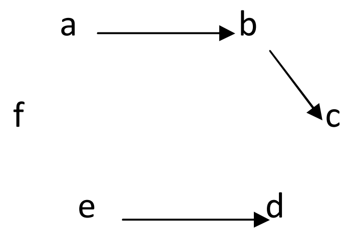
	In	out
a	0	1
b	1	1
c	1	1
d	1	0
e	0	0
f	0	0

$G^{2}_{6,3}$



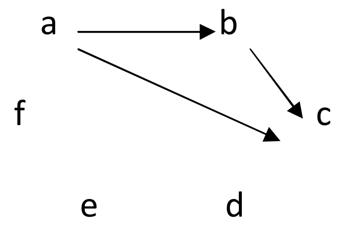
	In	out
a	0	1
b	1	1
c	2	0
d	0	1
e	0	0
f	0	0

$G^{3}_{6,3}$



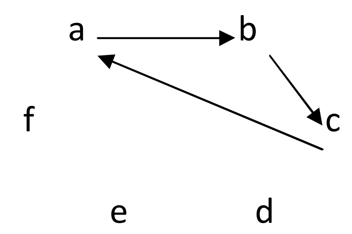
	In	out
a	0	1
b	1	1
c	1	0
d	1	0
e	0	1
f	0	0

$G^{4}_{6,3}$



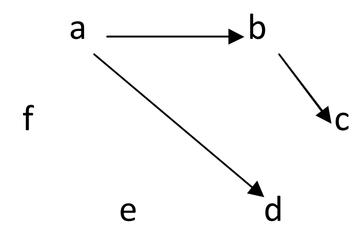
	In	out
a	0	2
b	1	1
c	2	0
d	0	0
e	0	0
f	0	0

$G^{5}_{6,3}$



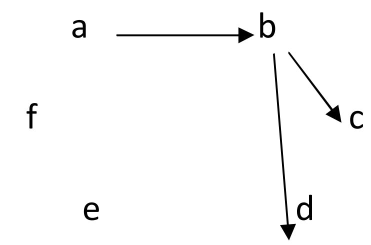
	In	out
a	1	1
b	1	1
c	1	1
d	0	0
e	0	0
f	0	0

$G^{6}_{6,3}$



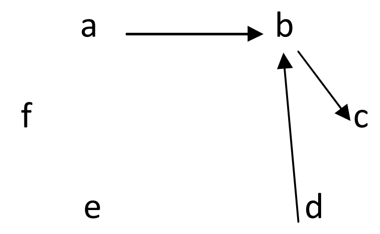
	In	out
a	0	2
b	1	1
c	1	0
d	1	0
e	0	0
f	0	0

$G^{7}_{6,3}$



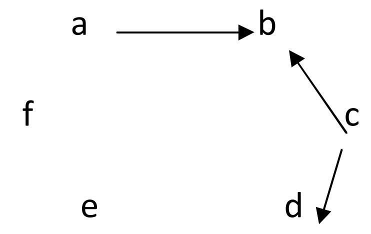
	In	out
a	0	1
b	1	2
c	1	0
d	1	0
e	0	0
f	0	0

$G^{8}_{6,3}$



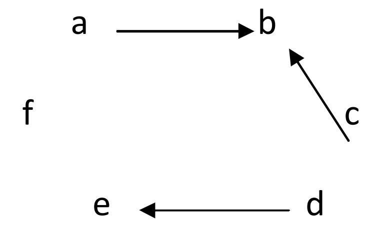
	In	out
a	0	1
b	2	1
c	1	0
d	0	1
e	0	0
f	0	0

$G^{9}_{6,3}$



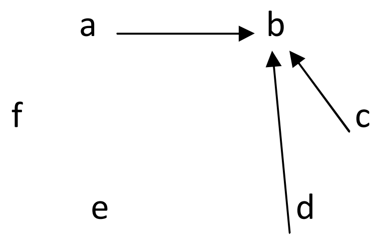
	In	out
a	0	1
b	2	0
c	0	2
d	1	0
e	0	0
f	0	0

$G^{10}_{6,3}$



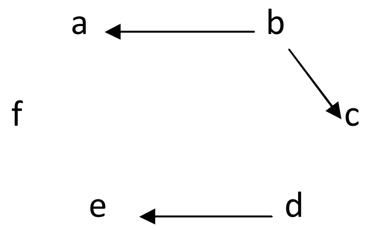
	In	out
a	0	1
b	2	0
c	0	1
d	0	1
e	1	0
f	0	0

$G^{11}_{6,3}$



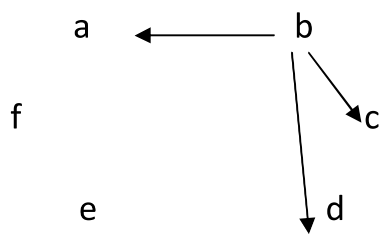
	In	out
a	0	1
b	3	0
c	0	1
d	0	1
e	0	0
f	0	0

$G^{12}_{6,3}$



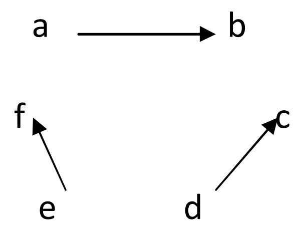
	In	out
a	1	0
b	0	2
c	1	0
d	0	1
e	1	0
f	0	0

$G^{13}_{6,3}$



	In	out
a	1	0
b	0	3
c	1	0
d	1	0
e	0	0
f	0	0

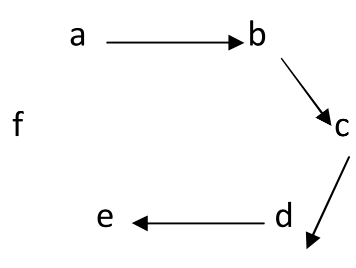
$G^{14}_{6,3}$



	In	out
a	0	1
b	1	0
c	1	0
d	0	1
e	0	1
f	1	0

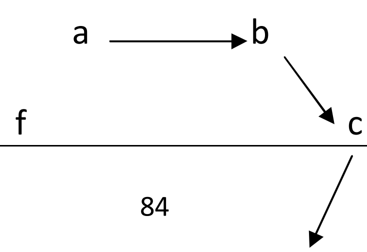
: $G_{6,4}$(5

$G^{1}_{6,4}$

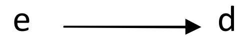
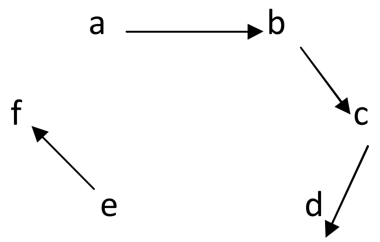
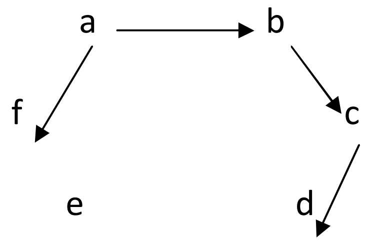
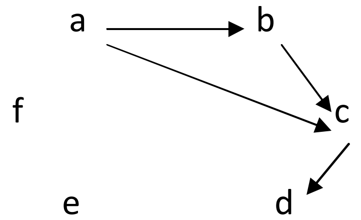
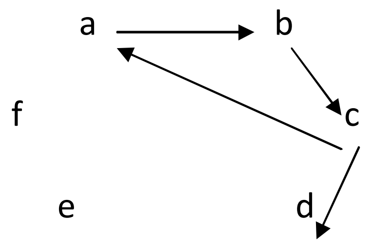
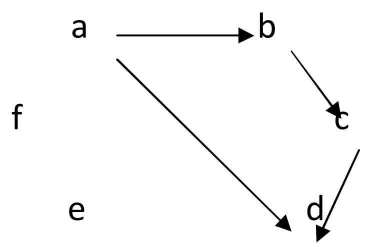


	In	out
a	0	1
b	1	1
c	1	1
d	1	1
e	1	0
f	0	0

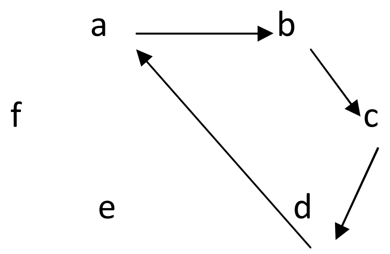
$G^{2}_{6,4}$



	In	out
a	0	1
b	1	1
c	1	1

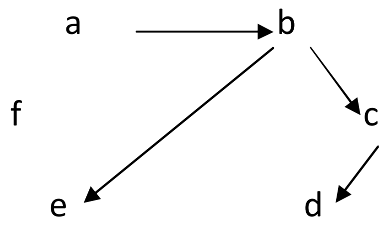
		d 2 0 e 0 1 f 0 0
$G^{3}_{6,4}$		In out a 0 1 b 1 1 c 1 1 d 1 0 e 0 1 f 1 0
$G^{4}_{6,4}$		In out a 0 2 b 1 1 c 1 1 d 1 0 e 0 0 f 1 0
$G^{5}_{6,4}$		In out a 0 2 b 1 1 c 2 1 d 1 0 e 0 0 f 0 0
$G^{6}_{6,4}$		In out a 1 1 b 1 1 c 1 2 d 1 0 e 0 0 f 0 0
$G^{7}_{6,4}$		In out a 0 2 b 1 1 c 1 1 d 2 0 e 0 0 f 0 0

$G^{8}_{6,4}$



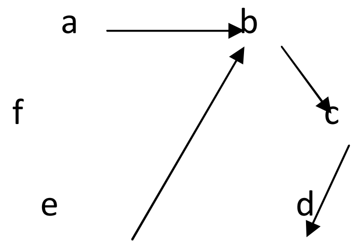
	In	out
a	1	1
b	1	1
c	1	1
d	1	1
e	0	0
f	0	0

$G^{9}_{6,4}$



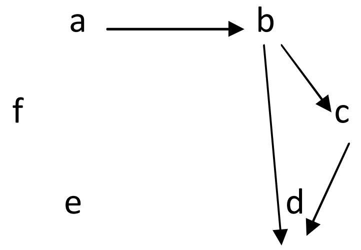
	In	out
a	0	1
b	1	2
c	1	1
d	1	0
e	1	0
f	0	0

$G^{10}_{6,4}$



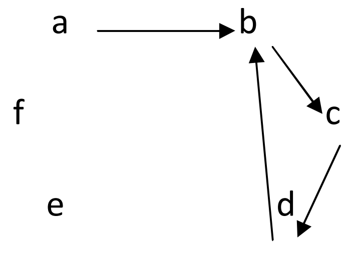
	In	out
a	0	1
b	2	1
c	1	1
d	1	0
e	0	1
f	0	0

$G^{11}_{6,4}$



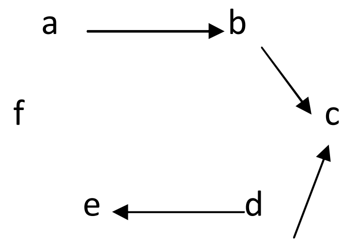
	In	out
a	0	1
b	1	2
c	1	1
d	2	0
e	0	0
f	0	0

$G^{12}_{6,4}$

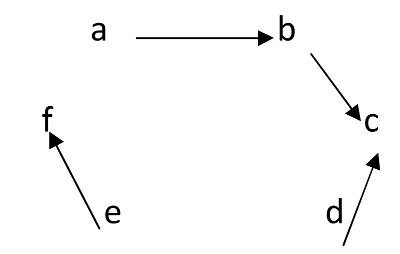
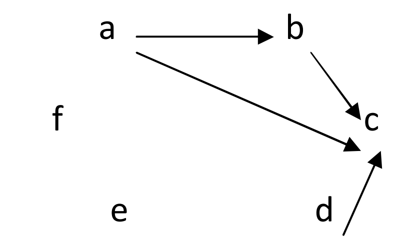
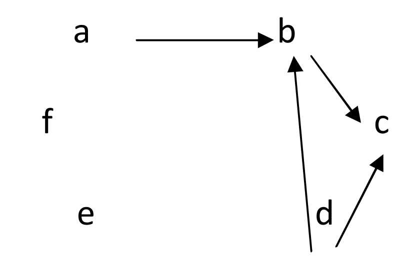
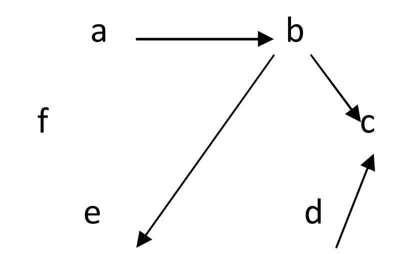
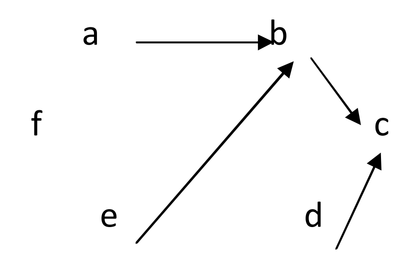
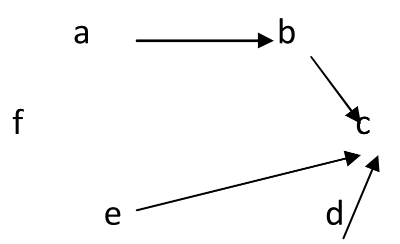


	In	out
a	0	1
b	2	1
c	1	1
d	1	1
e	0	0
f	0	0

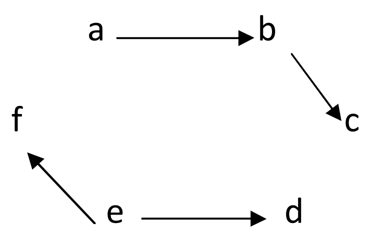
$G^{13}_{6,4}$



	In	out
a	0	1
b	1	1
c	2	0
d	0	2
e	1	0
f	0	0

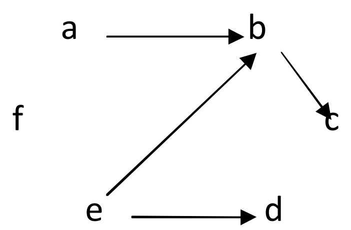
$G^{14}_{6,4}$		In out a 0 1 b 1 1 c 2 0 d 0 1 e 0 1 f 1 0
$G^{15}_{6,4}$		In out a 0 2 b 1 1 c 3 0 d 0 1 e 0 0 f 0 0
$G^{16}_{6,4}$		In out a 0 1 b 2 1 c 2 0 d 0 2 e 0 0 f 0 0
$G^{17}_{6,4}$		In out a 0 1 b 1 2 c 2 0 d 0 1 e 1 0 f 0 0
$G^{18}_{6,4}$		In out a 0 1 b 2 1 c 2 0 d 0 1 e 0 1 f 0 0
$G^{19}_{6,4}$		In out a 0 1 b 1 1 c 3 0 d 0 1 e 0 1 f 0 0

$G^{20}_{6,4}$



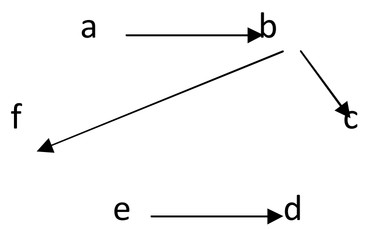
	In	out
a	0	1
b	1	1
c	1	0
d	1	0
e	0	2
f	1	0

$G^{21}_{6,4}$



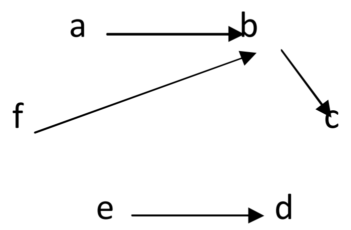
	In	out
a	0	1
b	2	1
c	1	0
d	1	0
e	0	2
f	0	0

$G^{22}_{6,4}$



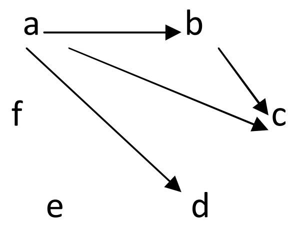
	In	out
a	0	1
b	1	2
c	1	0
d	1	0
e	0	1
f	1	0

$G^{23}_{6,4}$



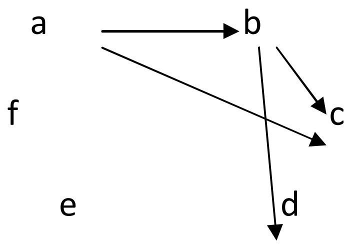
	In	out
a	0	1
b	2	1
c	1	0
d	1	0
e	0	1
f	0	1

$G^{24}_{6,4}$



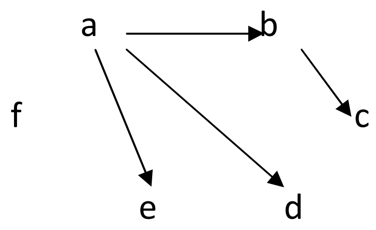
	In	out
a	0	3
b	1	1
c	2	0
d	1	0
e	0	0
f	0	0

$G^{25}_{6,4}$



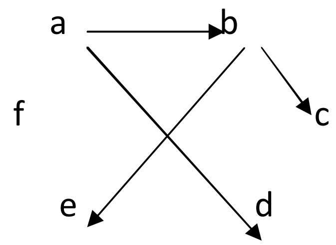
	In	out
a	0	2
b	1	2
c	2	0
d	1	0
e	0	0
f	0	0

$G^{26}_{6,4}$



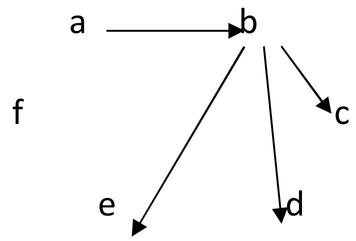
	In	out
a	0	3
b	1	1
c	1	1
d	1	0
e	1	0
f	0	0

$G^{27}_{6,4}$



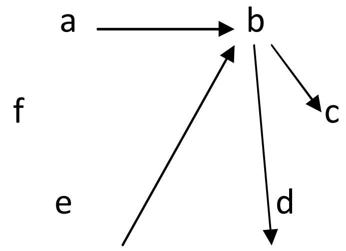
	In	out
a	0	2
b	1	2
c	1	0
d	1	0
e	1	0
f	0	0

$G^{28}_{6,4}$



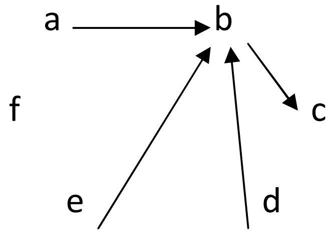
	In	out
a	0	1
b	1	3
c	1	0
d	1	0
e	1	0
f	0	0

$G^{29}_{6,4}$



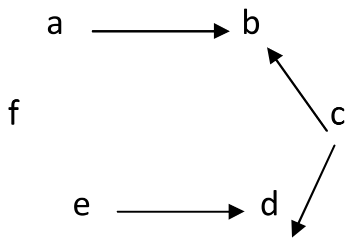
	In	out
a	0	1
b	2	2
c	1	0
d	1	0
e	0	1
f	0	0

$G^{30}_{6,4}$



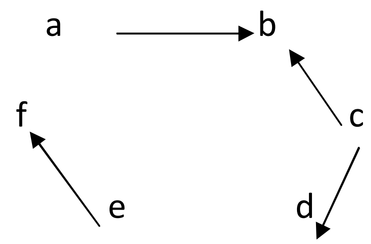
	In	out
a	0	1
b	3	1
c	1	0
d	0	1
e	0	1
f	0	0

$G^{31}_{6,4}$



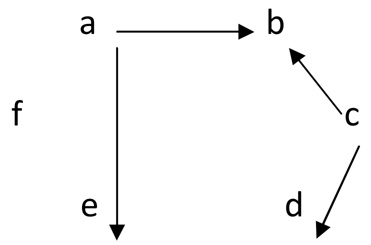
	In	out
a	0	1
b	2	0
c	0	2
d	2	0
e	0	1
f	0	0

$G^{32}_{6,4}$



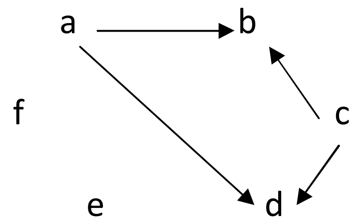
	In	out
a	0	1
b	2	0
c	0	2
d	1	0
e	0	1
f	1	0

$G^{33}_{6,4}$



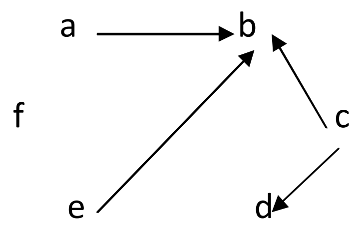
	In	out
a	0	2
b	2	0
c	0	2
d	1	0
e	1	0
f	0	0

$G^{34}_{6,4}$



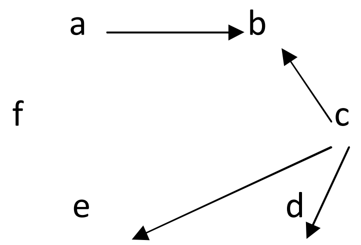
	In	out
a	0	2
b	2	0
c	0	2
d	2	0
e	0	0
f	0	0

$G^{35}_{6,4}$



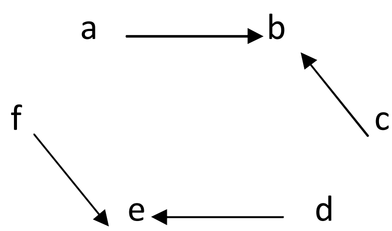
	In	out
a	0	1
b	3	0
c	0	2
d	1	0
e	0	1
f	0	0

$G^{36}_{6,4}$

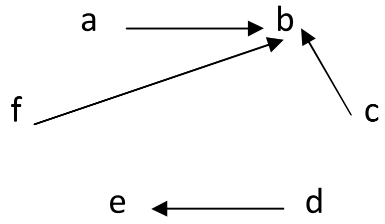
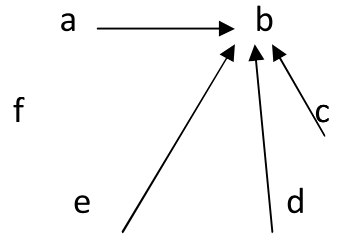
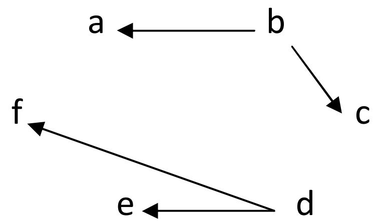
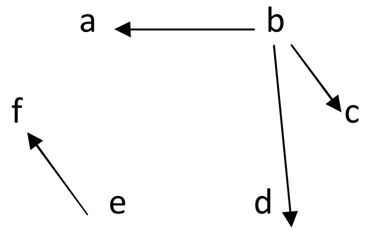
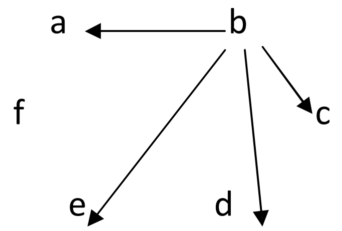


	In	out
a	0	1
b	2	0
c	0	3
d	1	0
e	1	0
f	0	0

$G^{37}_{6,4}$



	In	out
a	0	1
b	2	0
c	0	1
d	0	1
e	2	0
f	0	1

$G^{38}_{6,4}$		In out a 0 1 b 3 0 c 0 1 d 0 1 e 1 0 f 0 1
$G^{39}_{6,4}$		In out a 0 1 b 4 0 c 0 1 d 0 1 e 0 1 f 0 0
$G^{40}_{6,4}$		In out a 1 0 b 0 2 c 1 0 d 0 2 e 1 0 f 1 0
$G^{41}_{6,4}$		In out a 1 0 b 0 3 c 1 0 d 1 0 e 0 1 f 1 0
$G^{42}_{6,4}$		In out a 1 0 b 0 4 c 1 0 d 1 1 e 1 0 f 0 0

و نكتفي بهذا القدر من عرض البيانات المولدة بالمجموعة 6 مع التذكير بأن النتائج و عدد البيانات المتبقية مذكورة في جدول النتائج .

n \ m	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2	1	1														
3	1	1	3	2												
4	1	1	4	10	11	9	4									
5	1	1	4	13	33	53	72	87	62	31	9					
6	1	1	4	14	42	104	227	285	322	232	127	109	85	62	36	23

يمثل الجدول عدد البيانات $G_{m,n}$

حيث m عدد الأضلاع و n عدد الرؤوس .

الفصل الثالث

التشاكلات البيانية

نقوم في هذا الفصل بإيجاد التشاكلات البيانية للبيانات المولدة بالمجموعة 0,1,2,3,4

(3-1) التشاكلات البيانية :

لدينا نوعان من التشاكلات البيانية :

$$F: G_i \longrightarrow G_i$$

$G: G_i \longrightarrow G_j \quad : i \neq j$

سيتم ذكرها في الجداول التالية حيث أن O_i تمثل مجموعة رؤوس G_i ، A_i مجموعة أشعة G_i كما أن ab يرمز للضلع من a إلى b .

أمثلة :

1) التشاكل البياني $G^2_{3,2} \longrightarrow G^1_{2,1}$

موجود لأن $D_A: A^2_{3,2} \longrightarrow A^1_{2,1}$

$ab \longrightarrow ab$ ⇨ $(a \longrightarrow a)$, $(b \longrightarrow b)$

$cb \longrightarrow ab$ ⇨ $(c \longrightarrow a)$, $(b \longrightarrow b)$

أي أن عناصر $O_3 = \{a,b,c\}$ تنتقل إلى $O_2 = \{a,b\}$ وفق

$(a \longrightarrow a)$ ،$(b \longrightarrow b)$،$(c \longrightarrow a)$

2) التشاكل البياني $G^{11}_{4,3} \longrightarrow G^2_{3,3}$

موجود لأن $D_A: A^{11}_{4,3} \longrightarrow A^2_{3,3}$

$ad \longrightarrow ac$ ⇨ $(a \longrightarrow a)$, $(d \longrightarrow c)$

$ab \longrightarrow ab$ ⇨ $(a \longrightarrow a)$, $(b \longrightarrow b)$

$bc \longrightarrow bc$ ⇨ $(b \longrightarrow b)$, $(c \longrightarrow c)$

أي أن عناصر $O_4 = \{a,b,c,d\}$ تنتقل إلى $O_3 = \{a,b,c\}$ وفق

$(d \longrightarrow c)$، $(a \longrightarrow a)$ ،$(b \longrightarrow b)$،$(c \longrightarrow c)$

3) التشاكل البياني $G^2_{4,3} \longrightarrow G^1_{3,3}$

غير موجود لأن $D_A: A^2_{4,3} \longrightarrow A^1_{3,3}$

$ad \longrightarrow ca$ ⇨ $(a \longrightarrow c)$, $(d \longrightarrow a)$

$ab \longrightarrow ab$ ⇨ $(a \longrightarrow a)$, $(b \longrightarrow b)$

bc $\longrightarrow$ bc (b $\longrightarrow$ b), (c $\longrightarrow$ c)

لكن لا يمكن (c $\longrightarrow$ a), (a $\longrightarrow$ a) بنفس الوقت و بالتالي التشاكل غير موجود .

(3-1-1)التشاكلات البيانية بين G_0 - G_1 :

$G_0 \longrightarrow G_0$	$D_O : \emptyset \longrightarrow \emptyset$ $D_A : \emptyset \longrightarrow \emptyset$
$G_0 \longrightarrow G_1$	$D_O : \emptyset \longrightarrow O_1$ $D_A : \emptyset \longrightarrow \emptyset$
$G_1 \longrightarrow G_1$	$D_O : O_1 \longrightarrow O_1$ $D_A : \emptyset \longrightarrow \emptyset$

حيث $O_1 = \{ a \}$

(3-1-2)التشاكلات البيانية بين G_0 - G_1 - G_2 :

$G_0 \longrightarrow G_0$

$G_0 \longrightarrow G_0$	$D_O : \emptyset \longrightarrow \emptyset$ $D_A : \emptyset \longrightarrow \emptyset$

$G_0 \longrightarrow G_1$

$G_0 \longrightarrow G_1$	$D_O : \emptyset \longrightarrow O_1$ $D_A : \emptyset \longrightarrow \emptyset$

$G_0 \longrightarrow G_2$

$G_0 \longrightarrow G^1_{2,0}$	$D_O : \emptyset \longrightarrow O_2$ $D_A : \emptyset \longrightarrow \emptyset$

$G_1 \longrightarrow G_1$

$G_1 \longrightarrow G_1$	$D_O : O_1 \longrightarrow O_1$ $D_A : \emptyset \longrightarrow \emptyset$

$G_1 \longrightarrow G_2$

$G_1 \longrightarrow G^1_{2,0}$	$D_O : O_1 \longrightarrow O_2$ $D_A: \emptyset \longrightarrow \emptyset$

$G_2 \longrightarrow G_1$

$G^1_{2,0} \longrightarrow G_1$	$D_O : O_2 \longrightarrow O_1$ $D_A: \emptyset \longrightarrow \emptyset$

$G_2 \longrightarrow G_2$

$G^1_{2,0} \longrightarrow G^1_{2,0}$	$D_O : O_2 \longrightarrow O_2$ $D_A: \emptyset \longrightarrow \emptyset$
$G^1_{2,0} \longrightarrow G^1_{2,1}$	$D_O : O_2 \longrightarrow O_2$ $D_A: \emptyset \longrightarrow A_2$
$G^1_{2,1} \longrightarrow G^1_{2,1}$	$D_O : O_2 \longrightarrow O_2$, $a \longrightarrow a$, $b \longrightarrow b$ $D_A: A_2 \longrightarrow A_2$, $ab \longrightarrow ab$

حيث $O_1 = \{ a \}$ ، $O_2 = \{ a , b \}$

(3-1-3) التشاكلات البيانية بين G_0 - G_1 -G_2 -G_3 :

$G_0 \longrightarrow G_0$

$G_0 \longrightarrow G_0$	$D_O : \emptyset \longrightarrow \emptyset$ $D_A: \emptyset \longrightarrow \emptyset$

$G_0 \longrightarrow G_1$

$G_0 \longrightarrow G_1$	$D_O : \emptyset \longrightarrow O_1$ $D_A: \emptyset \longrightarrow \emptyset$

$G_0 \longrightarrow G_2$

$G_0 \longrightarrow G^1_{2,0}$	$D_O : \emptyset \longrightarrow O_2$ $\quad D_A : \emptyset \longrightarrow \emptyset$

$G_0 \longrightarrow G_3$

$G_0 \longrightarrow G^1_{3,0}$	$D_O : \emptyset \longrightarrow O_3$ $\quad D_A : \emptyset \longrightarrow \emptyset$

$G_1 \longrightarrow G_1$

$G_1 \longrightarrow G_1$	$D_O : O_1 \longrightarrow O_1$ $\quad D_A : \emptyset \longrightarrow \emptyset$

$G_1 \longrightarrow G_2$

$G_1 \longrightarrow G^1_{2,0}$	$D_O : O_1 \longrightarrow O_2$ $\quad D_A : \emptyset \longrightarrow \emptyset$

$G_1 \longrightarrow G_3$

$G_1 \longrightarrow G^1_{3,0}$	$D_O : O_1 \longrightarrow O_3$ $\quad D_A : \emptyset \longrightarrow \emptyset$

$G_2 \longrightarrow G_1$

$G^1_{2,0} \longrightarrow$ G1	$D_O : O_2 \longrightarrow O_1$ $\quad D_A : \emptyset \longrightarrow \emptyset$

$G_2 \longrightarrow G_2$

$G^1_{2,0} \longrightarrow G^1_{2,0}$	$D_O : O_2 \longrightarrow O_2$ $\quad D_A : \emptyset \longrightarrow \emptyset$

$G^1_{2,0} \longrightarrow G^1_{2,1}$	$D_O : O_2 \longrightarrow O_2$	$D_A: \emptyset \longrightarrow A_2$
$G^1_{2,1} \longrightarrow G^1_{2,1}$	$D_O : O_2 \longrightarrow O_2$ $a \longrightarrow a$ $b \longrightarrow b$	$D_A: A_2 \longrightarrow A_2$ $ab \longrightarrow ab$

$G_2 \longrightarrow G_3$

$G^1_{2,0} \longrightarrow G^1_{3,0}$	$D_O : O_2 \longrightarrow O_3$	$D_A: \emptyset \longrightarrow \emptyset$
$G^1_{2,0} \longrightarrow G^1_{3,1}$	$D_O : O_2 \longrightarrow O_3$	$D_A: \emptyset \longrightarrow A^1_{3,1}$
$G^1_{2,0} \longrightarrow G^1_{3,2}$	$D_O : O_2 \longrightarrow O_3$	$D_A: \emptyset \longrightarrow A^1_{3,2}$
$G^1_{2,0} \longrightarrow G^2_{3,2}$	$D_O : O_2 \longrightarrow O_3$	$D_A: \emptyset \longrightarrow A^2_{3,2}$
$G^1_{2,0} \longrightarrow G^3_{3,2}$	$D_O : O_2 \longrightarrow O_3$	$D_A: \emptyset \longrightarrow A^3_{3,2}$
$G^1_{2,0} \longrightarrow G^1_{3,3}$	$D_O : O_2 \longrightarrow O_3$	$D_A: \emptyset \longrightarrow A^1_{3,3}$
$G^1_{2,0} \longrightarrow G^2_{3,3}$	$D_O : O_2 \longrightarrow O_3$	$D_A: \emptyset \longrightarrow A^2_{3,3}$
$G^1_{2,1} \longrightarrow G^1_{3,1}$	$D_O : O_2 \longrightarrow O_3$ $a \longrightarrow a$ $b \longrightarrow b$	$D_A: A_2 \longrightarrow A^1_{3,1}$ $ab \longrightarrow ab$
$G^1_{2,1} \longrightarrow G^1_{3,2}$	$D_O : O_2 \longrightarrow O_3$ $a \longrightarrow a$ $b \longrightarrow b$	$D_A: A_2 \longrightarrow A^1_{3,2}$ $ab \longrightarrow ab$
	$D_O : O_2 \longrightarrow O_3$ $a \longrightarrow b$ $b \longrightarrow c$	$D_A: A_2 \longrightarrow A^1_{3,2}$ $ab \longrightarrow bc$
$G^1_{2,1} \longrightarrow G^2_{3,2}$	$D_O : O_2 \longrightarrow O_3$ $a \longrightarrow a$ $b \longrightarrow b$	$D_A: A_2 \longrightarrow A^2_{3,2}$ $ab \longrightarrow ab$

	$D_O: O_2 \rightarrow O_3$ $a \rightarrow c$ $b \rightarrow b$	$D_A: A_2 \rightarrow A^2_{3,2}$ $ab \rightarrow cb$
$G^1_{2,1} \longrightarrow G^3_{3,2}$	$D_O: O_2 \rightarrow O_3$ $a \rightarrow b$ $b \rightarrow a$	$D_A: A_2 \rightarrow A^3_{3,2}$ $ab \rightarrow bc$
	$D_O: O_2 \rightarrow O_3$ $a \rightarrow b$ $b \rightarrow c$	$D_A: A_2 \rightarrow A^3_{3,2}$ $ab \rightarrow bc$
$G^1_{2,1} \longrightarrow G^1_{3,3}$	$D_O: O_2 \rightarrow O_3$ $a \rightarrow a$ $b \rightarrow b$	$D_A: A_2 \rightarrow A^1_{3,3}$ $ab \rightarrow ab$
	$D_O: O_2 \rightarrow O_3$ $a \rightarrow b$ $b \rightarrow c$	$D_A: A_2 \rightarrow A^1_{3,3}$ $ab \rightarrow bc$
	$D_O: O_2 \rightarrow O_3$ $a \rightarrow c$ $b \rightarrow a$	$D_A: A_2 \rightarrow A^1_{3,3}$ $ab \rightarrow ca$
$G^1_{2,1} \longrightarrow G^2_{3,3}$	$D_O: O_2 \rightarrow O_3$ $a \rightarrow a$ $b \rightarrow b$	$D_A: A_2 \rightarrow A^2_{3,3}$ $ab \rightarrow ab$
	$D_O: O_2 \rightarrow O_3$ $a \rightarrow b$ $b \rightarrow c$	$D_A: A_2 \rightarrow A^2_{3,3}$ $ab \rightarrow bc$
	$D_O: O_2 \rightarrow O_3$ $a \rightarrow a$ $b \rightarrow c$	$D_A: A_2 \rightarrow A^2_{3,3}$ $ab \rightarrow ac$

$G_3 \longrightarrow G_1$

$G^1_{3,0} \longrightarrow G_1$	$D_O: O_3 \rightarrow O_1$	$D_A: \emptyset \rightarrow \emptyset$

$G_3 \longrightarrow G_2$

$G^1_{3,0} \longrightarrow G^1_{2,0}$	$D_O : O_3 \longrightarrow O_2$	$D_A : \emptyset \longrightarrow \emptyset$
$G^1_{3,0} \longrightarrow G^1_{2,1}$	$D_O : O_3 \longrightarrow O_2$	$D_A : \emptyset \longrightarrow A_2$
$G^1_{3,1} \longrightarrow G^1_{2,1}$	$D_O : O_3 \longrightarrow O_2$ $a \longrightarrow a$ $b \longrightarrow b$ $C \longrightarrow$	$D_A : A^1_{3,1} \longrightarrow A_2$ $ab \longrightarrow ab$ $bc \longrightarrow$
$G^2_{3,2} \longrightarrow G^1_{2,1}$	$D_O : O_3 \longrightarrow O_2$ $a \longrightarrow a$ $b \longrightarrow b$ $c \longrightarrow a$	$D_A : A^2_{3,2} \longrightarrow A_2$ $ab \longrightarrow ab$ $cb \longrightarrow ab$
$G^3_{3,2} \longrightarrow G^1_{2,1}$	$D_O : O_3 \longrightarrow O_2$ $a \longrightarrow b$ $b \longrightarrow a$ $c \longrightarrow b$	$D_A : A^3_{3,2} \longrightarrow A_2$ $ba \longrightarrow ab$ $bc \longrightarrow ab$

$G_3 \longrightarrow G_3$

$G^1_{3,0} \longrightarrow G^1_{3,0}$	$D_O : O_3 \longrightarrow O_3$	$D_A : \emptyset \longrightarrow \emptyset$
$G^1_{3,0} \longrightarrow G^1_{3,1}$	$D_O : O_3 \longrightarrow O_3$	$D_A : \emptyset \longrightarrow A^1_{3,1}$
$G^1_{3,0} \longrightarrow G^1_{3,2}$	$D_O : O_3 \longrightarrow O_3$	$D_A : \emptyset \longrightarrow A^1_{3,2}$
$G^1_{3,0} \longrightarrow G^2_{3,2}$	$D_O : O_3 \longrightarrow O_3$	$D_A : \emptyset \longrightarrow A^2_{3,2}$
$G^1_{3,0} \longrightarrow G^3_{3,2}$	$D_O : O_3 \longrightarrow O_3$	$D_A : \emptyset \longrightarrow A^3_{3,2}$
$G^1_{3,0} \longrightarrow G^1_{3,3}$	$D_O : O_3 \longrightarrow O_3$	$D_A : \emptyset \longrightarrow A^1_{3,3}$
$G^1_{3,0} \longrightarrow G^2_{3,3}$	$D_O : O_3 \longrightarrow O_3$	$D_A : \emptyset \longrightarrow A^2_{3,3}$
$G^1_{3,1} \longrightarrow G^1_{3,1}$	$D_O : O_3 \longrightarrow O_3$ $a \longrightarrow a$ $b \longrightarrow b$ $c \longrightarrow$	$D_A : A^1_{3,1} \longrightarrow A^1_{3,1}$ $ab \longrightarrow ab$
$G^1_{3,1} \longrightarrow G^1_{3,2}$	$D_O : O_3 \longrightarrow O_3$ $a \longrightarrow a$	$D_A : A^1_{3,1} \longrightarrow A^1_{3,2}$ $ab \longrightarrow ab$

	$b \to b$ $c \to$	
	$D_O : O_3 \to O_3$ $a \to b$ $b \to c$ $c \to$	$D_A: A_2 \to A^1_{3,2}$ $ab \to bc$
$G^1_{3,1} \to G^2_{3,2}$	$D_O : O_3 \to O_3$ $a \to a$ $b \to b$ $c \to$	$D_A: A^1_{3,1} \to A^2_{3,2}$ $ab \to ab$
	$D_O : O_3 \to O_3$ $a \to c$ $b \to b$ $c \to$	$D_A: A^1_{3,1} \to A^2_{3,2}$ $ab \to cb$
$G^1_{3,1} \to G^3_{3,2}$	$D_O : O_3 \to O_3$ $a \to b$ $b \to a$ $c \to$	$D_A: A^1_{3,1} \to A^3_{3,2}$ $ab \to ba$
	$D_O : O_3 \to O_3$ $a \to b$ $b \to c$ $c \to$	$D_A: A^1_{3,1} \to A^3_{3,2}$ $ab \to bc$
$G^1_{3,1} \to G^1_{3,3}$	$D_O : O_3 \to O_3$ $a \to a$ $b \to b$ $c \to$	$D_A: A^1_{3,1} \to A^1_{3,3}$ $ab \to ab$
	$D_O : O_3 \to O_3$ $a \to b$ $b \to c$ $c \to$	$D_A: A^1_{3,1} \to A^1_{3,3}$ $ab \to bc$
	$D_O : O_3 \to O_3$ $a \to c$ $b \to a$ $c \to$	$D_A: A^1_{3,1} \to A^1_{3,3}$ $ab \to ca$
$G^1_{3,1} \to G^2_{3,3}$	$D_O : O_3 \to O_3$ $a \to a$ $b \to b$ $c \to$	$D_A: A^1_{3,1} \to A^2_{3,3}$ $ab \to ab$
	$D_O : O_3 \to O_3$ $a \to b$ $b \to c$ $c \to$	$D_A: A^1_{3,1} \to A^2_{3,3}$ $ab \to bc$

	$D_O : O_3 \to O_3$ $a \to a$ $b \to c$ $c \to$	$D_A: A^1_{3,1} \to A^2_{3,3}$ $ab \to ac$
$G^1_{3,2} \to G^1_{3,2}$	$D_O : O_3 \to O_3$ $a \to a$ $b \to b$ $c \to c$	$D_A: A^1_{3,2} \to A^1_{3,2}$ $ab \to ab$ $bc \to bc$
$G^1_{3,2} \to G^1_{3,3}$	$D_O : O_3 \to O_3$ $a \to a$ $b \to b$ $c \to c$	$D_A: A^1_{3,2} \to A^1_{3,3}$ $ab \to ab$ $bc \to bc$
	$D_O : O_3 \to O_3$ $a \to b$ $b \to c$ $c \to a$	$D_A: A^1_{3,2} \to A^1_{3,3}$ $ab \to bc$ $bc \to ca$
	$D_O : O_3 \to O_3$ $a \to c$ $b \to a$ $c \to b$	$D_A: A^1_{3,2} \to A^1_{3,3}$ $ab \to ca$ $bc \to ab$
$G^1_{3,2} \to G^2_{3,3}$	$D_O : O_3 \to O_3$ $a \to a$ $b \to b$ $c \to c$	$D_A: A^1_{3,2} \to A^2_{3,3}$ $ab \to ab$ $bc \to bc$
$G^2_{3,2} \to G^1_{3,1}$	$D_O : O_3 \to O_3$ $a \to a$ $b \to b$ $c \to a$	$D_A: A^2_{3,2} \to A^1_{3,1}$ $ab \to ab$ $cb \to ab$
$G^2_{3,2} \to G^2_{3,2}$	$D_O : O_3 \to O_3$ $a \to a$ $b \to b$ $c \to c$	$D_A: A^2_{3,2} \to A^2_{3,2}$ $ab \to ab$ $cb \to cb$
$G^2_{3,2} \to G^2_{3,3}$	$D_O : O_3 \to O_3$ $a \to b$ $b \to c$ $c \to a$	$D_A: A^2_{3,2} \to A^2_{3,3}$ $ab \to bc$ $cb \to ac$
$G^3_{3,2} \to G^1_{3,1}$	$D_O : O_3 \to O_3$ $a \to b$ $b \to a$ $c \to b$	$D_A: A^3_{3,2} \to A^1_{3,1}$ $ba \to ab$ $bc \to ab$

$G^{3}_{3,2} \longrightarrow G^{3}_{3,2}$	$D_O : O_3 \longrightarrow O_3$ $a \longrightarrow a$ $b \longrightarrow b$ $c \longrightarrow c$	$D_A: A^{3}_{3,2} \longrightarrow A^{3}_{3,2}$ $ba \longrightarrow ba$ $bc \longrightarrow bc$
$G^{3}_{3,2} \longrightarrow G^{2}_{3,3}$	$D_O : O_3 \longrightarrow O_3$ $a \longrightarrow b$ $b \longrightarrow a$ $c \longrightarrow c$	$D_A: A^{3}_{3,2} \longrightarrow A^{2}_{3,3}$ $ba \longrightarrow ab$ $bc \longrightarrow ac$
$G^{1}_{3,3} \longrightarrow G^{1}_{3,3}$	$D_O : O_3 \longrightarrow O_3$ $a \longrightarrow a$ $b \longrightarrow b$ $c \longrightarrow c$	$D_A: A^{1}_{3,3} \longrightarrow A^{1}_{3,3}$ $ab \longrightarrow ab$ $bc \longrightarrow bc$ $ac \longrightarrow ca$
$G^{2}_{3,3} \longrightarrow G^{2}_{3,3}$	$D_O : O_3 \longrightarrow O_3$ $a \longrightarrow a$ $b \longrightarrow b$ $c \longrightarrow c$	$D_A: A^{2}_{3,3} \longrightarrow A^{2}_{3,3}$ $ab \longrightarrow ab$ $bc \longrightarrow bc$ $ac \longrightarrow ac$

وأدرجنا النتائج بالشكل التالي :

	G_0	G_1	G_2	G_3	G_4
G_0	1	1	1	1	1
G_1	0	1	1	1	1

G_2	0	1	3	13	81
G_3	0	1	5	24	199
G_4	0	1	10	49	469

الفصل الرابع

ايجاد التشاكلات الفئوية المثبة للعناصر(Fixob)

للفئات $Grph_{<n}$ من أجل $n\geq 0$:

نقوم في هذا الفصل بتوصيف الفئات $Grph_{<n}$: $n=1,2,3,4,5$

ومن ثم ايجاد التشاكلات الفئوية المثبتة للعناصر (Fixob) لكل منها .

تعريف:

لتكن A فئة ما ، نقول عن التشاكل الفئوي $F: A \longrightarrow A$

إنه تشاكل مثبت للعناصر (Fixob) إذا كانت صورة أي عنصر a من A هو نفسه
أي $F(a) = a$ وذلك من أجل كل a من A

اصطلاحات :

- ليكن $F : Grph_{<n} \longrightarrow Grph_{<n}$

و f شعاعا ما من $Grph_{<n}$ نقول إن f شعاع مثبت وفق F إذا كان $F(f)=f$ Fixob

و نقول إن الشعاع f غير مثبت وفق F إذا كان $F(f)\neq f$.

- إذا كان g_i أحد أشعة $Grph_{<n}$ و لنفرض أن $G^k_{n,m}$ $g_i : G^k_{n,m}$

عندئذ نرمز g^j_i (J=1،2 ،...... ،σ) لجميع التشاكلات البيانية بين البيانين $G^k_{n,m}$ ، $G^k_{n,m}$

و بالتالي سنطلق على g^X_i الاختيار أو الخيار x للتشاكل البياني g_i .

(1-4)الفئات $Grph_{<n}$:

: $Grph_{<1}$

- ❖ عناصرها G_0
- ❖ أشعتها :شعاع واحد

$f: G_0 \longrightarrow G_0$ $D_O:\emptyset \longrightarrow \emptyset$ $DA:\emptyset \longrightarrow \emptyset$

: $Grph_{<2}$

- ❖ عناصرها G_0 - G_1
- ❖ أشعتها : (3) أشعة

$f_1 : G_0 \longrightarrow G_0$ $D_O:\emptyset \longrightarrow \emptyset$ $D_A:\emptyset \longrightarrow \emptyset$

$f_2: G_0 \longrightarrow G_1 \quad D_O: \emptyset \longrightarrow O_1 \quad D_A: \emptyset \longrightarrow \emptyset$

$f_3: G_1 \longrightarrow G_1 \quad D_O: O_1 \longrightarrow O_1 \quad D_A: \emptyset \longrightarrow \emptyset$

: $Grph_{<3}$

عناصرها : G_0 - G_1 - $G^1_{2,0}$ - $G^1_{2,1}$

❖ أشعتها :

$f_1: G_0 \longrightarrow G_0 \quad D_O: \emptyset \longrightarrow \emptyset \quad D_A: \emptyset \longrightarrow \emptyset$

$f_2: G_0 \longrightarrow G_1 \quad D_O: \emptyset \longrightarrow O_1 \quad D_A: \emptyset \longrightarrow \emptyset$

$f_3: G_0 \longrightarrow G^1_{2,0} \quad D_O: \emptyset \longrightarrow O_2 \quad D_A: \emptyset \longrightarrow \emptyset$

$f_4: G_1 \longrightarrow G_1 \quad D_O: O_1 \longrightarrow O_1 \quad D_A: \emptyset \longrightarrow \emptyset$

$f_5: G^1_{2,0} \longrightarrow G^1_{2,0} \quad D_O: O_2 \longrightarrow O_2 \quad D_A: \emptyset \longrightarrow \emptyset$

$f_6: G^1_{2,0} \longrightarrow G^1_{2,1} \quad D_O: O_2 \longrightarrow O_2 \quad D_A: \emptyset \longrightarrow A_2$

$f_7: G^1_{2,1} \longrightarrow G^1_{2,1} \quad D_o: O_2 \longrightarrow O_2 \quad D_A: A_2 \longrightarrow A_2$

$a \longrightarrow a \quad (ab) \longrightarrow (ab)$

$b \longrightarrow b$

$f_8: G^1_{2,0} \longrightarrow G_1 \quad D_O: O_2 \longrightarrow O_1 \quad D_A: \emptyset \longrightarrow \emptyset$

$f_9: G_1 \longrightarrow G^1_{2,0} \quad D_O: O_1 \longrightarrow O_2 \quad D_A: \emptyset \longrightarrow \emptyset$

: $Grph_{<4}$

❖ عناصرها

$- G^{1}_{3,3} - G^{3}_{3,2} - G^{2}_{3,2} - G^{1}_{3,2} - G^{1}_{3,1} - G^{1}_{2,1} - G^{1}_{3,0} - G^{1}_{2,0} -. G_1 - G_0$

$G^{2}_{3,3,}$

❖ أشعتها (54) شعاعا . (موضحة في الفقرة (3-1-3))

: $Grph_{<5}$

❖ عناصرها

$-- G^{2}_{3,3} - G^{1}_{3,3} - G^{3}_{3,2} - G^{2}_{3,2} - G^{1}_{3,2} - G^{1}_{3,1} - G^{1}_{2,1} - G^{1}_{3,0} - G^{1}_{2,0} -- G_1 -- G_0$

$- G^{3}_{4,3} - G^{2}_{4,3} - G^{4}_{4,2} - G^{1}_{4,3} - G^{3}_{4,2} - G^{2}_{4,2} - G^{1}_{4,2} - G^{1}_{4,1} - G^{1}_{3,0} - G^{1}_{4,0}$

$- G^{3}_{4,4} - G^{2}_{4,4} - G^{1}_{4,4,} - G^{10}_{4,3} - G^{9}_{4,3} - G^{8}_{4,3,} - G^{7}_{4,3} - G^{6}_{4,3},- G^{5}_{4,3} - G^{4}_{4,3,}$

$- G^{2}_{4,5} \; G^{1}_{4,5} - G^{11}_{4,4} -- G^{10}_{4,4,} - G^{9}_{4,4} - G^{8}_{4,4} - G^{7}_{4,4},- G^{6}_{4,4} - G^{5}_{4,4} - G^{4}_{4,4,}$

$G^{4}_{4,6} - G^{3}_{4,6} - G^{2}_{4,6} - G^{1}_{4,6} - G^{9}_{4,5} - G^{8}_{4,5} - G^{7}_{4,5} - G^{6}_{4,5} - G^{5}_{4,5} - G^{4}_{4,5} - G^{3}_{4,5} -$

. أشعتها : - (865) شعاعا

و لم يذكر جميع هذه الأشعة في متن هذه الأطروحة لكبر عددها و لعدم الحاجة إليها في البراهين القادمة .

(2-4) التشاكلات الفئوية المثبتة للعناصر (Fixob)

(1-2-4)إيجاد Fixob لأجل $Grph_{<1}$:

بما أن $Grph_{<1}$ له شعاع واحد مثبت وفق Fفإن **Fixob** هو التشاكل الفئوي الواحدي .

(2-2-4)إيجاد Fixob لأجل $Grph_{<2}$:

بما أن جميع الأشعة هي أشعة مثبتة و فق F **فإن Fixob** هو التشاكل الفئوي الواحدي ..

(3-2-4) إيجاد Fixob لأجل $Grph_{<3}$:

بماأن جميع الأشعة هي أشعة مثبتة و فق Fفإن Fixob هو التشاكل الفئوي الواحدي .

(4-2-4) إيجاد Fixob لأجل $Grph_{<4}$:

: $Grph_{<4}$

❖ عناصرها

$G^{2}_{3,3},\ - G^{1}_{3,3} - G^{3}_{3,2} - G^{2}_{3,2} - G^{1}_{3,2} - G^{1}_{3,1} - G^{1}_{2,1} - G^{1}_{3,0} - G^{1}_{2,0} -. G_{1} - G_{0}$

❖ أشعتها :(54) شعاع

نلاحظ أن : عددها (43) شعاع أي أن $F(f_i) = f_i$ و هي :

- $D_A: \emptyset \longrightarrow \emptyset$ $\quad D_O: \emptyset \longrightarrow \emptyset$ $\quad f_1: G_0 \longrightarrow G_0$
- $D_A: \emptyset \longrightarrow \emptyset$ $\quad D_O: \emptyset \longrightarrow O_1$ $\quad f_2: G_0 \longrightarrow G_1$
- $D_A: \emptyset \longrightarrow \emptyset$ $\quad D_O: \emptyset \longrightarrow O_2$ $\quad f_3: G_0 \longrightarrow G^{1}_{2,0}$
- $D_A: \emptyset \longrightarrow \emptyset$ $\quad D_O: \emptyset \longrightarrow O_3$ $\quad f_4: G_0 \longrightarrow G^{1}_{3,0}$
- $D_A: \emptyset \longrightarrow \emptyset$ $\quad D_O: O_2 \longrightarrow O_1$ $\quad f_5: G^{1}_{2,0} \longrightarrow G_1$
- $D_A: \emptyset \longrightarrow \emptyset$ $\quad D_O: O_2 \longrightarrow O_2$ $\quad f_6: G^{1}_{2,0} \longrightarrow G^{1}_{2,0}$
- $D_A: \emptyset \longrightarrow A_2$ $\quad D_O: O_2 \longrightarrow O_2$ $\quad f_7: G^{1}_{2,0} \longrightarrow G^{1}_{2,1}$
- $D_A: \emptyset \longrightarrow \emptyset$ $\quad D_O: O_2 \longrightarrow O_3$ $\quad f_8: G^{1}_{2,0} \longrightarrow G^{1}_{3,0}$
- $D_A: \emptyset \longrightarrow A^{1}_{3,1}$ $\quad D_O: O_2 \longrightarrow O_3$ $\quad f_9: G^{1}_{2,0} \longrightarrow G^{1}_{3,1}$
- $D_A: \emptyset \longrightarrow A^{1}_{3,2}$ $\quad D_O: O_2 \longrightarrow O_3$ $\quad f_{10}: G^{1}_{2,0} \longrightarrow G^{1}_{3,2}$

$f_{11}: G^{1}_{2,0} \longrightarrow G^{2}_{3,2}$ $D_O:O_2 \longrightarrow O_3$ $D_A:\emptyset \longrightarrow A^{2}_{3,2}$ •

$f_{12}: G^{1}_{2,0} \longrightarrow G^{3}_{3,2}$ $D_O:O_2 \longrightarrow O_3$ $D_A:\emptyset \longrightarrow A^{3}_{3,2}$ •

$f_{13}: G^{1}_{2,0} \longrightarrow G^{1}_{3,3}$ $D_O:O_2 \longrightarrow O_3$ $D_A:\emptyset \longrightarrow A^{1}_{3,3}$ •

$f_{14}: G^{1}_{2,0} \longrightarrow G^{2}_{3,3}$ $D_O:O_2 \longrightarrow O_3$ $D_A:\emptyset \longrightarrow A^{2}_{3,3}$ •

$f_{15}: G^{1}_{3,0} \longrightarrow G_1$ $D_O:O_3 \longrightarrow O_1$ $D_A:\emptyset \longrightarrow \emptyset$ •

$f_{16}: G^{1}_{3,0} \longrightarrow G^{1}_{2,0}$ $D_O:O_3 \longrightarrow O_2$ $D_A:\emptyset \longrightarrow \emptyset$ •

$f_{17}: G^{1}_{3,0} \longrightarrow G^{1}_{2,1}$ $D_O:O_3 \longrightarrow O_2$ $D_A:\emptyset \longrightarrow A_2$ •

$f_{18}: G^{1}_{3,0} \longrightarrow G^{1}_{3,0}$ $D_O:O_3 \longrightarrow O_3$ $D_A:\emptyset \longrightarrow \emptyset$ •

$f_{19}: G^{1}_{3,0} \longrightarrow G^{1}_{3,1}$ $D_O:O_3 \longrightarrow O_3$ $D_A:\emptyset \longrightarrow A^{1}_{3,1}$ •

$f_{20}: G^{1}_{3,0} \longrightarrow G^{1}_{3,2}$ $D_O:O_3 \longrightarrow O_3$ $D_A:\emptyset \longrightarrow A^{1}_{3,2}$ •

$f_{21}: G^{1}_{3,0} \longrightarrow G^{2}_{3,2}$ $D_O:O_3 \longrightarrow O_3$ $D_A:\emptyset \longrightarrow A^{2}_{3,2}$ •

$f_{22}: G^{1}_{3,0} \longrightarrow G^{3}_{3,2}$ $D_O:O_3 \longrightarrow O_3$ $D_A:\emptyset \longrightarrow A^{3}_{3,2}$ •

$f_{23}: G^{1}_{3,0} \longrightarrow G^{1}_{3,3}$ $D_O: O_3 \longrightarrow O_3$ $D_A: \emptyset \longrightarrow A^{1}_{3,3}$ •

$f_{24}: G^{1}_{3,0} \longrightarrow G^{2}_{3,3}$ $D_O: O_3 \longrightarrow O_3$ $D_A: \emptyset \longrightarrow A^{2}_{3,3}$ •

$f_{25}: G_1 \longrightarrow G_1$ $D_O: O_2 \longrightarrow O_2$ $D_A: \emptyset \longrightarrow \emptyset$ •

$f_{26}: G^{1}_{2,1} \longrightarrow G^{1}_{2,1}$ $D_o: O_2 \longrightarrow O_2$ $D_A: A^{1}_{2,1} \longrightarrow A^{1}_{2,1}$ •

$a \longrightarrow a$ $(ab) \longrightarrow (ab)$ •

$b \longrightarrow b$

$f_{27}: G^{1}_{2,1} \longrightarrow G^{1}_{3,1}$ $D_o: O_2 \longrightarrow O_3$ $D_A: A^{1}_{2,1} \longrightarrow A^{1}_{3,1}$

$a \longrightarrow a$ $(ab) \longrightarrow (ab)$ •

$b \longrightarrow b$

$f_{28}: G^{1}_{3,1} \longrightarrow G^{1}_{2,1}$ $D_o: O_3 \longrightarrow O_2$ $D_A: A^{1}_{3,1} \longrightarrow A^{1}_{2,1}$

$a \longrightarrow a$ $(ab) \longrightarrow (ab)$ •

$b \longrightarrow b$

$f_{29}: G^{2}_{3,2} \longrightarrow G^{1}_{2,1}$ $D_o: O_3 \longrightarrow O_2$ $D_A: A^{2}_{3,2} \longrightarrow A^{1}_{2,1}$

$a \longrightarrow a$ $(ab) \longrightarrow (ab)$ •

$b \longrightarrow b$ $(cb) \longrightarrow (ab)$

$c \longrightarrow a$

$f_{30}: G^{3}_{3,2} \longrightarrow G^{1}_{2,1}$ $D_o: O_3 \longrightarrow O_2$ $D_A: A^{3}_{3,2} \longrightarrow A^{1}_{2,1}$

	$a \longrightarrow b$	$(ba) \longrightarrow (ab)$ •
	$b \longrightarrow a$	$(bc) \longrightarrow (ab)$
	$c \longrightarrow b$	

$f_{31}: G^{1}_{3,1} \longrightarrow G^{1}_{3,1}$	$D_o: O_3 \longrightarrow O_3$	$D_A: A^{1}_{3,1} \longrightarrow A^{1}_{3,1}$
	$a \longrightarrow a$	$(ab) \longrightarrow (ab)$ •
	$b \longrightarrow b$	

$f_{32}: G^{1}_{3,2} \longrightarrow G^{1}_{3,2}$	$D_o: O_3 \longrightarrow O_3$	$D_A: A^{1}_{3,2} \longrightarrow A^{1}_{3,2}$
	$a \longrightarrow a$	$(ab) \longrightarrow (ab)$ •
	$b \longrightarrow b$	$(bc) \longrightarrow (bc)$
	$c \longrightarrow c$	

$f_{33}: G^{1}_{3,2} \longrightarrow G^{2}_{3,3}$	$D_o: O_3 \longrightarrow O_3$	$D_A: A^{1}_{3,2} \longrightarrow A^{2}_{3,3}$
	$a \longrightarrow a$	$(ab) \longrightarrow (ab)$ •
	$b \longrightarrow b$	$(bc) \longrightarrow (bc)$
	$c \longrightarrow c$	

$f_{34}: G^{2}_{3,2} \longrightarrow G^{1}_{3,1}$	$D_o: O_3 \longrightarrow O_3$	$D_A: A^{2}_{3,2} \longrightarrow A^{1}_{3,1}$
	$a \longrightarrow a$	$(ab) \longrightarrow (ab)$ •
	$b \longrightarrow b$	$(cb) \longrightarrow (ab)$
	$c \longrightarrow a$	

$f_{35}: G^{2}_{3,2} \longrightarrow G^{2}_{3,2}$	$D_o: O_3 \longrightarrow O_3$	$D_A: A^{2}_{3,2} \longrightarrow A^{2}_{3,2}$	
	$a \longrightarrow a$	$(ab) \longrightarrow (ab)$	•
	$b \longrightarrow b$	$(cb) \longrightarrow (cb)$	
	$c \longrightarrow c$		

$f_{36}: G^{3}_{3,2} \longrightarrow G^{1}_{3,1}$	$D_o: O_3 \longrightarrow O_3$	$D_A: A^{3}_{3,2} \longrightarrow A^{1}_{3,1}$	
	$a \longrightarrow b$	$(ba) \longrightarrow (ab)$	
	$b \longrightarrow a$	$(bc) \longrightarrow (ab)$	
	$c \longrightarrow b$		

$f_{37}: G^{3}_{3,2} \longrightarrow G^{3}_{3,2}$	$D_o: O_3 \longrightarrow O_3$	$D_A: A^{3}_{3,2} \longrightarrow A^{3}_{3,2}$	
	$a \longrightarrow a$	$(ba) \longrightarrow (ba)$	•
	$b \longrightarrow b$	$(bc) \longrightarrow (bc)$	
	$c \longrightarrow c$		

$f_{38}: G^{3}_{3,2} \longrightarrow G^{2}_{3,3}$	$D_o: O_3 \longrightarrow O_3$	$D_A: A^{3}_{3,2} \longrightarrow A^{2}_{3,3}$	
	$a \longrightarrow b$	$(ba) \longrightarrow (ab)$	•
	$b \longrightarrow a$	$(bc) \longrightarrow (ac)$	
	$c \longrightarrow c$		

$f_{39}: G^{1}_{3,3} \longrightarrow G^{1}_{3,3}$	$D_o: O_3 \longrightarrow O_3$	$D_A: A^{1}_{3,3} \longrightarrow A^{1}_{3,3}$	
	$a \longrightarrow a$	$(ab) \longrightarrow (ab)$	•
	$b \longrightarrow b$	$(bc) \longrightarrow (bc)$	
	$c \longrightarrow c$	$(ca) \longrightarrow (ca)$	

- $f_{40}: G^{2}_{3,3} \longrightarrow G^{2}_{3,3}$ $D_o: O_3 \longrightarrow O_3$ $D_A: A^2_{3,3} \longrightarrow A^2_{3,3}$

$a \longrightarrow a$ $(ab) \longrightarrow (ab)$

$b \longrightarrow b$ $(bc) \longrightarrow (bc)$

$c \longrightarrow c$ $(ac) \longrightarrow (ac)$

- $f_{41}: G^{2}_{3,2} \longrightarrow G^{2}_{3,3}$ $D_o: O_3 \longrightarrow O_3$ $D_A: A^2_{3,2} \longrightarrow A^2_{3,3}$

$a \longrightarrow b$ $(ab) \longrightarrow (bc)$

$b \longrightarrow c$ $(cb) \longrightarrow (ac)$

$c \longrightarrow a$

- $f_{42}: G_1 \longrightarrow G^{1}_{2,0}$ $D_O: O_1 \longrightarrow O_2$ $D_A: \emptyset \longrightarrow \emptyset$

- $f_{43}: G_1 \longrightarrow G^{1}_{3,0}$ $D_O: O_1 \longrightarrow O_3$ $D_A: \emptyset \longrightarrow \emptyset$

بينما الأشعة التالية و التي لها عدة احتمالات نبينها فيما يلي :

1) $g_1: G^{1}_{2,1} \longrightarrow G^{1}_{3,2}$

لها حالتين :

- $g^1_1: G^{1}_{2,1} \longrightarrow G^{1}_{3,2}$ - $g^2_1: G^{1}_{2,1} \longrightarrow G^{1}_{3,2}$

- $D_O: O_2 \longrightarrow O_3$ $D_O: O_2 \longrightarrow O_3$

- $a \longrightarrow a$ $a \longrightarrow b$

$b \longrightarrow b$ $b \longrightarrow c$

أي $F(g^1_1) = g^1_1$ أو g^2_1 - $F(g^2_1) = g^1_1$ أو g^2_1

2) $g_2: G^{1}_{2,1} \longrightarrow G^{2}_{3,2}$

لها حالتين : -

- $g^1_2: G^{1}_{2,1} \longrightarrow G^{2}_{3,2}$ - $g^2_2: G^{1}_{2,1} \longrightarrow G^{2}_{3,2}$

$D_O : O_2 \longrightarrow O_3$ - $D_O : O_2 \longrightarrow O_3$

$a \longrightarrow a$ - $a \longrightarrow c$

$b \longrightarrow b$ $b \longrightarrow b$

أي $F(g^1_2) = g^1_2$ أو g^2_2 - $F(g^2_2) = g^1_2$ أو g^2_2

3) $g_3 : G^1_{2,1} \longrightarrow G^3_{3,2}$

لها حالتين :

- $g^1_3 : G^1_{2,1} \longrightarrow G^3_{3,2}$ - $g^2_3 : G^1_{2,1} \longrightarrow G^3_{3,2}$

$D_O : O_2 \longrightarrow O_3$ $D_O : O_2 \longrightarrow O_3$

$a \longrightarrow b$ $a \longrightarrow b$

$b \longrightarrow a$ $b \longrightarrow c$

أي $F(g^1_3) = g^1_3$ أو g^2_3 - $F(g^2_3) = g^1_3$ أو g^2_3

4) $g_4 : G^1_{2,1} \longrightarrow G^1_{3,3}$

لها 3حالات :

$g^1_4 : G^1_{2,1} \longrightarrow G^1_{3,3}$ - $g^2_4 : G^1_{2,1} \longrightarrow G^1_{3,3}$ - $g^3_4 : G^1_{2,1} \longrightarrow G^1_{3,3}$

$D_O : O_2 \longrightarrow O_3$ $D_O : O_2 \longrightarrow O_3$ $D_O : O_2 \longrightarrow O_3$

$a \longrightarrow a$ $a \longrightarrow b$ $a \longrightarrow c$

$b \longrightarrow b$ $b \longrightarrow c$ $b \longrightarrow a$

أي $F(g^1_4) = g^1_4$ أو g^2_4 أو g^3_4 - $F(g^2_4) = g^1_4$ أو g^2_4 أو g^2_4

- $F(g^3_4) = g^1_4$ أو g^2_4 أو g^3_4

5) $g_5 : G^1_{2,1} \longrightarrow G^2_{3,3}$

لها 3حالات :

$g^1_5 : G^1_{2,1} \longrightarrow G^2_{3,3}$ - $g^2_5 : G^1_{2,1} \longrightarrow G^2_{3,3}$ - $g^3_5 : G^1_{2,1} \longrightarrow G^2_{3,3}$

$D_O : O_2 \longrightarrow O_3$ $D_O : O_2 \longrightarrow O_3$ $-D_O : O_2 \longrightarrow O_3$

$a \longrightarrow a$ $a \longrightarrow b$ $a \longrightarrow a$

$b \longrightarrow b$ $b \longrightarrow c$ $b \longrightarrow c$

أي $F(g^1_5) = g^1_5$ أو g^2_5 أو g^3_5 - $F(g^2_5) = g^1_5$ أو g^2_5 أو g^2_5

- $F(g^3_5) = g^1_5$ أو g^2_5 أو g^3_5

6) $g_6 : G^1_{3,1} \longrightarrow G^1_{3,2}$

لها حالتين :

- $g^1_6 : G^1_{3,1} \longrightarrow G^1_{3,2}$

$D_O : O_3 \longrightarrow O_3$

$a \longrightarrow a$

$b \longrightarrow b$

$g^2_6 : G^1_{3,1} \longrightarrow G^1_{3,2}$

$D_O : O_3 \longrightarrow O_3$

$a \longrightarrow b$

$b \longrightarrow c$

أي $F(g^1_6) = g^1_6$ أو g^2_6 - $F(g^2_6) = g^1_6$ أو 6_1

7) $g_7 : G^1_{3,1} \longrightarrow G^2_{3,2}$

لها حالتين :

- $g^1_7 : G^1_{3,1} \longrightarrow G^2_{3,2}$ -

$D_O : O_3 \longrightarrow O_3$

$a \longrightarrow a$

$b \longrightarrow b$

$g^2_7 : G^1_{3,1} \longrightarrow G^2_{3,2}$

$D_O : O_3 \longrightarrow O_3$

$a \longrightarrow c$

$b \longrightarrow b$

أي $F(g^1_7) = g^1_7$ أو g^2_7 - $F(g^2_7) = g^1_7$ أو g^2_7

8) $g_8 : G^1_{3,1} \longrightarrow G^3_{3,2}$

لها حالتين :

- $g^1_8 : G^1_{3,1} \longrightarrow G^3_{3,2}$

$D_O : O_3 \longrightarrow O_3$

- $a \longrightarrow b$

$b \longrightarrow a$

$g^2_8 : G^1_{3,1} \longrightarrow G^3_{3,2}$

$D_O : O_3 \longrightarrow O_3$

$a \longrightarrow b$

$b \longrightarrow c$

أي $F(g^1_8) = g^1_8$ أو g^2_8 - $F(g^2_8) = g^1_8$ أو g^2_8

9) $g_9 : G^1_{3,1} \longrightarrow G^1_{3,3}$

لها 3حالات :

$g^1_9 : G^1_{3,1} \longrightarrow G^1_{3,3}$ - $g^2_9 : G^1_{3,1} \longrightarrow G^1_{3,3}$ - $g^3_9 : G^1_{3,1} \longrightarrow G^1_{3,3}$

g^3_9		g^2_9		g^1_9	
$D_O : O_3$	O_3	$D_O : O_3$	O_3	$D_O : O_3$	O_3
a	c	a	b	a	a
b	a	b	c	b	b

أي $F(g^1_9) = g^1_9$ أو g^2_9 أو g^3_9 - $F(g^2_9) = g^1_9$ أو g^2_9 أو g^2_9

- $F(g^3_9) = g^1_9$ أو g^2_9 أو g^3_9

10) $g_{10}: G^1_{3,1} \longrightarrow G^2_{3,3}$

لها 3حالات :

$g^1_{10} : G^1_{2,1} \longrightarrow G^1_{3,3}$ - $g^2_{10} : G^1_{2,1} \longrightarrow G^1_{3,3}$ - $g^3_{10} : G^1_{2,1} \longrightarrow G^1_{3,3}$

g^3_{10}		g^2_{10}		g^1_{10}	
$D_O : O_3$	O_3	$D_O : O_3$	O_3	$D_O : O_3$	O_3
a	a	a	b	a	a
b	c	b	c	b	b

أي $F(g^1_{10}) = g^1_{10}$ أو g^2_{10} أو g^3_{10} - $F(g^2_{10}) = g^1_{10}$ أو g^2_{10} أو g^2_{10}

- $F(g^3_{10}) = g^1_{10}$ أو g^2_{10} أو g^3_{10}

11) $g_{11}: G^1_{3,2} \longrightarrow G^1_{3,3}$

لها 3حالات :

$g^1_{11} : G^1_{3,2} \longrightarrow G^1_{3,3}$ - $g^2_{11} : G^1_{3,2} \longrightarrow G^1_{3,3}$ - $g^3_{11} : G^1_{3,2} \longrightarrow G^1_{3,3}$

g^3_{11}		g^2_{11}		g^1_{11}	
$D_O : O_3$	O_3	$D_O : O_3$	O_3	$D_O : O_3$	O_3
a	c	a	b	a	a
b	a	b	c	b	b
c	b	c	a	c	c

أي $F(g^1_{11}) = g^1_{11}$ أو g^2_{11} أو g^3_{11} - $F(g^2_{11}) = g^1_{11}$ أو g^2_{11} أو g^2_{11}

- $F(g^3_{11}) = g^1_{11}$ أو g^2_{11} أو g^3_{11}

$$F: Grph_{<4} \longrightarrow Grph_{<4}$$

و بما أنF لا يثبت جميع الأشعة فيمكن أن يعرف fixob غير الواحدي و لكن قبل الجزم بذلك لا بد من التحقق من أن F يحقق شروط التشاكل الفئوي .

لذلك قمنا بإيجاد جميع التراكيب الممكنة بين الأشعة المثبتة و الغير مثبتة وفق F و التي عددها (27) تركيب

❖ $f_{27}of_{26} : G^1_{2,1} \longrightarrow G^1_{2,1} \longrightarrow G^1_{3,1}$

$F(f_{27}) \; o \; F(f_{26}) = f_{27}of_{26} = f_{27}$ ، $F(f_{27}of_{26}) = F(f_{27}) = f_{27}$

و بالتالي $F(f_{27}of_{26}) = F(f_{27}) \; o \; F(f_{26})$

❖ $f_{26}of_{28} : G^1_{3,1} \longrightarrow G^1_{2,1} \longrightarrow G^1_{2,1}$

$F(f_{26}of_{28}) = F(f_{28}) = f_{28}$،$F(f_{26}) \; o \; F(f_{28}) = f_{26}of_{28} = f_{28}$

و بالتالي $F(f_{26}of_{28}) = F(f_{26}) \; o \; F(f_{28})$

❖ $f_{28}of_{27} : G^1_{2,1} \longrightarrow G^1_{3,1} \longrightarrow G^1_{2,1}$

$F(f_{28}) \; o \; F(f_{27}) = f_{28}of_{27} = f_{26}$ ،$F(f_{28}of_{27}) = F(f_{26}) = f_{26}$

و بالتالي $F(f_{34}of_{35}) = F(f_{34}) \; o \; F(f_{35})$

❖ $f_{26}of_{29} : G^2_{3,2} \longrightarrow G^1_{2,1} \longrightarrow G^1_{2,1}$

$F(f_{26}) \; o \; F(f_{29}) = f_{26}of_{29} = f_{29}$ ، $F(f_{26}of_{29}) = F(f_{29}) = f_{29}$

وبالتالي $F(f_{26}of_{29}) = F(f_{26}) \; o \; F(f_{29})$

❖ $f_{27}of_{29} : G^2_{3,2} \longrightarrow G^1_{2,1} \longrightarrow G^1_{3,1}$

$F(f_{27}) \; o \; F(f_{29}) = f_{27}of_{29} = f_{34}$ ، $F(f_{27}of_{29}) = F(f_{34}) = f_{34}$

و بالتالي $F(f_{27}of_{29}) = F(f_{27}) \; o \; F(f_{29})$

❖ $f_{26}of_{30} : G^{3}_{3,2} \longrightarrow G^{1}_{2,1} \longrightarrow G^{1}_{2,1}$

$F(f_{26}) \circ F(f_{30}) = f_{26}of_{30} = f_{30}$ ، $F(f_{26}of_{30}) = F(f_{30}) = f_{30}$

و بالتالي $F(f_{26}of_{30}) = F(f_{26}) \circ F(f_{30})$

❖ $f_{27}of_{30} : G^{3}_{3,2} \longrightarrow G^{1}_{2,1} \longrightarrow G^{1}_{3,1}$

$F(f_{27}) \circ F(f_{30}) = f_{27}of_{30} = f_{36}$ ، $F(f_{27}of_{30}) = F(f_{36}) = f_{36}$

وبالتالي $F(f_{27}of_{30}) = F(f_{27}) \circ F(f_{30})$

❖ $f_{28}of_{34} : G^{2}_{3,2} \longrightarrow G^{1}_{3,1} \longrightarrow G^{1}_{2,1}$

$F(f_{28}) \circ F(f_{34}) = f_{28}of_{34} = f_{29}$ ، $F(f_{28}of_{34}) = F(f_{29}) = f_{29}$

و بالتالي $F(f_{28}of_{34}) = F(f_{28}) \circ F(f_{34})$

❖ $f_{28}of_{36} : G^{3}_{3,2} \longrightarrow G^{1}_{3,1} \longrightarrow G^{1}_{2,1}$

$F(f_{28}) \circ F(f_{36}) = f_{28}of_{36} = f_{30}$ ، $F(f_{28}of_{36}) = F(f_{30}) = f_{30}$

و بالتالي $F(f_{28}of_{36}) = F(f_{28}) \circ F(f_{36})$

❖ $f_{33}og_{1}: G^{1}_{2,1} \longrightarrow G^{1}_{3,2} \longrightarrow G^{2}_{3,3}$

• $f_{33}: G^{1}_{3,2} \longrightarrow G^{2}_{3,3}$ $D_o: O_3 \longrightarrow O_3$ $D_A: A^{1}_{3,2} \longrightarrow A^{2}_{3,3}$

$a \longrightarrow a$ $(ab) \longrightarrow (ab)$

$b \longrightarrow b$ $(bc) \longrightarrow (bc)$

$c \longrightarrow c$

1) لأجل $F(g^{1}_{1}) = g^{1}_{1}$ حيث $f_{33}og^{1}_{1} = g^{1}_{5}$

- $F(f_{33}) \circ F(g^{1}_{1})(a) = F(f_{33})(F(g^{1}_{1})(a)) = F(f_{33})(g^{1}_{1}(a) = F(f_{33})(a) = a$

- $F(f_{33}og^1{}_1)(a) = F(g^1{}_5)(a) = a$

- $F(f_{33}) \, o \, F(g^1{}_1)(b) = F(f_{33})(F(g^1{}_1)(b)) = F(f_{33})(g^1{}_1(b) = F(f_{33})(b) = b$

- ومنه $F(f_{33}og^1{}_1)(b) = F(g^1{}_5)(b) = b$

وبالتالي يجب أن يكون $F(g^1{}_5) = g^1{}_5$

2) لأجل $F(g^1{}_1) = g^2{}_1$

- $F(f_{33}) \, o \, F(g^1{}_1)(a) = F(f_{33})(F(g^1{}_1)(a)) = F(f_{33})(g^2{}_1(a)) = F(f_{33})(b) = b$

- ومنه $F(f_{33}og^1{}_1)(a) = F(g^1{}_5)(a) = b$

$F(f_{33}) \, o \, F(g^1{}_1)(b) = F(f_{33})(F(g^1{}_1)(b)) = F(f_{33})(g^2{}_1(b)) = F(f_{33})(c) = c$

- ومنه $F(f_{33}og^1{}_1)(b) = F(g^1{}_5)(b) = c$

وبالتالي يجب أن يكون $F(g^1{}_5) = g^2{}_5$

3) **لأجل $F(g^2{}_1) = g^1{}_1$ حيث $f_{33}og^2{}_1 = g^2{}_5$**

- $F(f_{33}) \, o \, F(g^2{}_1)(a) = F(f_{33})(F(g^2{}_1)(a)) = F(f_{33})(g^1{}_1(a)) = F(f_{33})(a) = a$

- ومنه $F(f_{33}og^2{}_1)(a) = F(g^2{}_5)(a) = a$

- $F(f_{33}) \, o \, F(g^2{}_1)(b) = F(f_{33})(F(g^2{}_1)(b)) = F(f_{33})(g^1{}_1(b)) = F(f_{33})(b) = b$

- ومنه $F(f_{33}og^2{}_1)(b) = F(g^2{}_5)(b) = b$

وبالتالي يجب أن يكون $F(g^2{}_5) = g^1{}_5$

4) **لأجل $F(g^2{}_1) = g^2{}_1$**

- $F(f_{33}) \, o \, F(g^2{}_1)(a) = F(f_{33})(F(g^2{}_1)(a)) = F(f_{33})(g^2{}_1(a)) = F(f_{33})(b) = b$

- ومنه $F(f_{33}og^2{}_1)(a) = F(g^2{}_5)(a) = b$

$F(f_{33}) \, o \, F(g^2{}_1)(b) = F(f_{33})(F(g^2{}_1)(b)) = F(f_{33})(g^2{}_1(b)) = F(f_{33})(c) = c$

- ومنه $F(f_{33}og^2{}_1)(b) = F(g^2{}_5)(b) = c$

وبالتالي يجب أن يكون $F(g^2{}_5) = g^2{}_5$

❖ $f_{41}og_2: \quad G^1_{2,1} \longrightarrow G^2_{3,2} \longrightarrow G^2_{3,3}$

$f_{41}: G^2_{3,2} \longrightarrow G^2_{3,3}$ $\quad D_o: O_3 \longrightarrow O_3$ $\quad D_A: A^2_{3,2} \longrightarrow A^2_{3,3}$

a	→ b	(ab)	→ (bc)	•
b	→ c	(cb)	→ (ac)	
c	→ a			

1) لأجل $F(g^1_2) = g^1_2$ حيث $f_{41}og^1_2 = g^2_5$

- $F(f_{41})\ o\ F(g^1_2)\ (a) = F\ (f_{41})\ (\ F(g^1_2)\ (a)\ ) = F(f_{41})(\ g^1_2(a) = F(f_{41})\ (a) = b$
- ومنه $F(f_{41}og^1_2)\ (a) = F(\ g^2_5\ )\ (a) = b$
- $F(f_{41})\ o\ F(g^1_2)\ (b) = F\ (f_{41})\ (\ F(g^1_2)\ (b)\ ) = F(f_{41})(\ g^1_2(b) = F(f_{41})(b) = c$
- ومنه $F(f_{41}og^1_2)\ (b) = F(\ g^2_5\ )\ (b) = c$
- وبالتالي يجب أن يكون $F(g^2_5) = g^2_5$

2) لأجل $F(g^1_2) = g^2_2$

- $F(f_{41})\ o\ F(g^1_2)\ (a) = F\ (f_{41})\ (\ F(g^1_2)\ (a)\ ) = F(f_{41})(\ g^2_2(a)) = F(f_{41})\ (c) = a$
- ومنه $F(f_{41}og^1_2)\ (a) = F(\ g^1_5\ )\ (a) = a$

$F(f_{41})\ o\ F(g^1_2)\ (b) = F\ (f_{41})\ (\ F(g^1_2)\ (b)\ ) = F(f_{41})(\ g^2_2(b)\ ) = F(f_{41})(b) = c$

- ومنه $F(f_{41}og^1_2)\ (b) = F(\ g^1_5\ )\ (b) = c$

وبالتالي يجب أن يكون $F(g^2_5) = g^3_5$

$f_{41}og^2_2 = g^3_5$ **حيث** $F(g^2_2) = g^1_2$ **لأجل** (3

- $F(f_{41})\ o\ F(g^2_2)(a) = F(f_{41})(F(g^2_2)(a)) = F(f_{41})(g^1_2(a)) = F(f_{41})(a) = b$
- ومنه $F(f_{41}og^2_2)(a) = F(g^3_5)(a) = b$
- $F(f_{41})\ o\ F(g^2_2)(b) = F(f_{41})(F(g^2_2)(b)) = F(f_{41})(g^1_2(b)) = F(f_{41})(b) = c$
- ومنه $F(f_{41}og^2_2)(b) = F(g^3_5)(b) = c$

وبالتالي يجب أن يكون $F(g^3_5) = g^2_5$

$F(g^2_2) = g^2_2$ **لأجل** (4

- $F(f_{41})\ o\ F(g^2_2)(a) = F(f_{41})(F(g^2_2)(a)) = F(f_{41})(g^2_2(a)) = F(f_{41})(c) = a$
- ومنه $F(f_{41}og^2_2)(a) = F(g^3_5)(a) = a$
- $F(f_{41})\ o\ F(g^2_2)(b) = F(f_{41})(F(g^2_2)(b)) = F(f_{41})(g^2_2(b)) = F(f_{41})(b) = c$

ومنه $F(f_{41}og^2_2)(b) = F(g^3_5)(b) = c$

وبالتالي يجب أن يكون $F(g^3_5) = g^3_5$

❖ $$f_{38}og_3: \quad G^1_{2,1} \longrightarrow G^3_{3,2} \longrightarrow G^2_{3,3}$$

$f_{38}: G^3_{3,2} \longrightarrow G^2_{3,3}$ $D_o: O_3 \longrightarrow O_3$ $D_A: A^3_{3,2} \longrightarrow A^2_{3,3}$

D_o		D_A	
a	b	(ba)	(ab)
b	a	(bc)	(ac)
c	c		

إن $f_{38}og^1_3 = g^1_5$

1) لأجل $F(g^1_3) = g^1_3$

- $F(f_{38}) \circ F(g^1_3)(a) = F(f_{38})(F(g^1_3)(a)) = F(f_{38})(g^1_3(a) = F(f_{38})(a) = a$
- ومنه $F(f_{38} o g^1_3)(a) = F(g^1_5)(a) = a$

$F(f_{38}) \circ F(g^1_3)(b) = F(f_{38})(F(g^1_3)(b)) = F(f_{38})(g^1_3(b) = F(f_{38})(a) = b$

- ومنه $F(f_{38} o g^1_3)(b) = F(g^1_5)(b) = b$
- وبالتالي يجب أن يكون $F(g^1_5) = g^1_5$

2) لأجل $F(g^1_3) = g^2_3$

- $F(f_{38}) \circ F(g^1_3)(a) = F(f_{38})(F(g^1_3)(a)) = F(f_{38})(g^2_3(a)) = F(f_{38})(b) = a$
- ومنه $F(f_{38} o g^1_3)(a) = F(g^1_5)(a) = a$

$F(f_{38}) \circ F(g^1_3)(b) = F(f_{38})(F(g^1_3)(b)) = F(f_{38})(g^2_3(b)) = F(f_{38})(c) = c$

- ومنه $F(f_{38} o g^1_3)(b) = F(g^1_5)(b) = c$

وبالتالي يجب أن يكون $F(g^1_5) = g^3_5$

3) لأجل $F(g^2_3) = g^1_3$ حيث $f_{38} o g^2_3 = g^3_5$

- $F(f_{38}) \circ F(g^2_3)(a) = F(f_{38})(F(g^2_3)(a)) = F(f_{38})(g^1_3(a)) = F(f_{38})(b) = a$
- ومنه $F(f_{38} o g^2_3)(a) = F(g^3_5)(a) = a$

$F(f_{38}) \circ F(g^2_3)(b) = F(f_{38})(F(g^2_3)(b)) = F(f_{38})(g^1_3(b)) = F(f_{38})(a) = b$

- ومنه $F(f_{38} o g^2_3)(b) = F(g^3_5)(b) = b$

وبالتالي يجب أن يكون $F(g^3_5) = g^1_5$

4) لأجل $F(g^2_3) = g^2_3$

- $F(f_{38}) \circ F(g^2_3)(a) = F(f_{38})(F(g^2_3)(a)) = F(f_{38})(g^2_3(a)) = F(f_{38})(b) = a$
- ومنه $F(f_{38} o g^2_3)(a) = F(g^3_5)(a) = a$

$F(f_{38}) \circ F(g^2_3)(b) = F(f_{38})(F(g^2_3)(b)) = F(f_{38})(g^2_3(b)) = F(f_{38})(c) = c$

- ومنه $F(f_{38} o g^2_3)(b) = F(g^3_5)(b) = c$

وبالتالي يجب أن يكون $F(g^3_5) = g^3_5$

❖ $g_1 o f_{28}$: $G^1_{3,1} \longrightarrow G^1_{2,1} \longrightarrow G^1_{3,3}$

$f_{28}: G^1_{3,1} \longrightarrow G^1_{2,1}$ $D_o: O_3 \longrightarrow O_2$ $D_A: A^1_{3,1} \longrightarrow A^1_{2,1}$

- $(ab) \longrightarrow (ab)$ $a \longrightarrow a$

$b \longrightarrow b$

إن $g_1 o f_{28} = g^1_6$

1) لأجل $F(g^1_1) = g^1_1$

- $F(g^1_1) \circ F(f_{28})(a) = F(g^1_1)(F(f_{28})(a)) = F(g^1_1)(a) = g^1_1(a) = a$
- ومنه $F(g^1_1 o f_{28})(a) = F(g^1_6)(a) = a$
- $F(g^1_1) \circ F(f_{28})(b) = F(g^1_1)(F(f_{28})(b)) = F(g^1_1(b) = g^1_1(b) = b$
- ومنه $F(g^1_1 o f_{28})(b) = F(g^1_6)(b) = b$
- وبالتالي يجب أن يكون $F(g^1_6) = g^1_6$

2) لأجل $F(g^1_1) = g^2_1$

- $F(g^1_1) \circ F(f_{28})(a) = F(g^1_1)(F(f_{28})(a)) = F(g^1_1(a)) = g^2_1(a) = b$
- ومنه $F(g^1_1 o f_{28})(a) = F(g^1_6)(a) = b$

$F(g^1_1) \circ F(f_{28})(b) = F(g^1_1)(F(f_{28})(b)) = F(g^1_1(b)) = g^2_1(b) = c$

- ومنه $F(g^1_1 o f_{28})(b) = F(g^1_6)(b) = c$

وبالتالي يجب أن يكون $F(g^1_6) = g^2_6$

(3 لأجل $F(g^2_1) = g^1_1$ حيث $g^2_1 o f_{28} = g^2_6$

- $F(g^2_1)\ o\ F(f_{28})(a) = F(g^2_1)(F(f_{28})(a)) = F(g^2_1(a)) = g^1_1(a) = a$
- ومنه $F(g^2_1 o f_{28})(a) = F(g^2_6)(a) = a$

$F(g^2_1)\ o\ F(f_{28})(b) = F(g^2_1)(F(f_{28})(b)) = F(g^2_1(b)) = g^1_1(b) = b$

- ومنه $F(g^2_1 o f_{28})(b) = F(g^2_6)(b) = b$

وبالتالي يجب أن يكون $F(g^2_6) = g^1_6$

(4 لأجل $F(g^2_1) = g^2_1$

- $F(g^2_1)\ o\ F(f_{28})(a) = F(g^2_1)(F(f_{28})(a)) = F(g^2_1(a)) = g^2_1(a) = b$
- ومنه $F(g^2_1 o f_{28})(a) = F(g^2_6)(a) = b$
- $F(g^2_1)\ o\ F(f_{28})(b) = F(g^2_1)(F(f_{28})(b)) = F(g^2_1(b)) = g^2_1(b) = c$

ومنه $F(g^2_1 o f_{28})(b) = F(g^2_6)(b) = c$

وبالتالي يجب أن يكون $F(g^2_6) = g^2_6$

❖ **$g_2 o f_{28}$: $G^1_{3,1} \longrightarrow G^1_{2,1} \longrightarrow G^2_{3,2}$**

• $f_{28}: G^1_{3,1} \longrightarrow G^1_{2,1}$ $\quad D_o: O_3 \longrightarrow O_2$ $\quad D_A: A^1_{3,1} \longrightarrow A^1_{2,1}$

$a \longrightarrow a$ $\quad (ab) \longrightarrow (ab)$

$b \longrightarrow b$

❖ إن $g^1_2 o f_{28} = g^1_7$

(1 لأجل $F(g^1_2) = g^1_2$

- $F(g^1{}_2) \circ F(f_{28})(a) = F(g^1{}_2)(F(f_{28})(a)) = F(g^1{}_2)(f_{28}(a) = F(g^1{}_2)(a) = a$

- $F(g^1{}_2 o f_{28})(a) = F(g^1{}_7)(a) = a$

- $F(g^1{}_2) \circ F(f_{28})(b) = F(g^1{}_2)(F(f_{28})(b)) = F(g^1{}_2)(f_{28}(b) = (g^1{}_2)(b) = b$

$F(g^1{}_2 o f_{28})(b) = F(g^1{}_7)(b) = b$

يجب أن يكون $F(g^1{}_7) = g^1{}_7$

2) لأجل $F(g^1{}_2) = g^2{}_2$

- $F(g^1{}_2) \circ F(f_{28})(a) = F(g^1{}_2)(F(f_{28})(a)) = F(g^1{}_2(a)) = g^2{}_2(a) = c$

- ومنه $F(g^1{}_2 o f_{28})(a) = F(g^1{}_7)(a) = c$

$F(g^1{}_2) \circ F(f_{28})(b) = F(g^1{}_2)(F(f_{28})(b)) = F(g^1{}_2)(b)) = g^2{}_2(b) = b$

ومنه $F(g^1{}_2 o f_{28})(b) = F(g^1{}_7)(b) = b$

وبالتالي يجب أن يكون $F(g^1{}_7) = g^2{}_7$

3) لأجل $F(g^2{}_2) = g^1{}_2$ حيث $g^2{}_2 o f_{28} = g^2{}_7$

- $F(g^2{}_2) \circ F(f_{28})(a) = F(g^2{}_2)(F(f_{28})(a)) = F(g^2{}_2)(a)) = g^1{}_2(a) = a$

- ومنه $F(g^2{}_2 o f_{28})(a) = F(g^2{}_7)(a) = a$

- $F(g^2{}_2) \circ F(f_{28})(b) = F(g^2{}_2)(F(f_{28})(b)) = F(g^2{}_2)(b)) = g^1{}_2(b) = b$

- ومنه $F(g^2{}_2 o f_{28})(b) = F(g^2{}_7)(b) = b$

وبالتالي يجب أن يكون

$F(g^2{}_7) = g^1{}_7$

4) لأجل $F(g^2{}_2) = g^2{}_2$

- $F(g^2{}_2) \circ F(f_{28})(a) = F(g^2{}_2)(F(f_{28})(a)) = F(g^2{}_2)(a)) = g^2{}_2(a) = c$

- ومنه $F(g^2{}_2 o f_{28})(a) = F(g^2{}_7)(a) = c$

$F(g^2{}_2) \circ F(f_{28})(b) = F(g^2{}_2)(F(f_{28})(b)) = F(g^2{}_2)(b)) = g^2{}_2(b) = b$

- ومنه $F(g^2{}_2 o f_{28})(b) = F(g^2{}_7)(b) = b$

وبالتالي يجب أن يكون $F(g^2{}_7) = g^2{}_7$

❖ **$g_3 o f_{28}$: $G^1{}_{3,1} \longrightarrow G^1{}_{2,1} \longrightarrow G^3{}_{3,2}$**

$f_{28}: G^1{}_{3,1} \longrightarrow G^1{}_{2,1}$

$D_o: O_3 \longrightarrow O_2$

$a \longrightarrow a$

$b \longrightarrow b$

$D_A: A^1{}_{3,1} \longrightarrow A^1{}_{2,1}$

• $(ab) \longrightarrow (ab)$

إن $g^1{}_3 o f_{28} = g^1{}_8$

1) لأجل $F(g^1{}_3) = g^1{}_3$

- $F(g^1{}_3) \circ F(f_{28})(a) = F(g^1{}_3)(F(f_{28})(a)) = F(g^1{}_3)(a) = g^1{}_3(a) = b$
- ومنه $F(g^1{}_3 o f_{28})(a) = F(g^1{}_8)(a) = b$

$F(g^1{}_3) \circ F(f_{28})(b) = F(g^1{}_3)(F(f_{28})(b)) = F(g^1{}_3)(b) = g^1{}_3(b) = a$

- ومنه $F(g^1{}_3 o f_{28})(b) = F(g^1{}_8)(b) = a$
- وبالتالي يجب أن يكون $F(g^1{}_8) = g^1{}_8$

2) لأجل $F(g^1{}_3) = g^2{}_3$

- $F(g^1{}_3) \circ F(f_{28})(a) = F(g^1{}_3)(F(f_{28})(a)) = F(g^1{}_3)(a)) = g^2{}_3(a) = b$
- ومنه $F(g^1{}_3 o f_{28})(a) = F(g^1{}_8)(a) = b$
- $F(g^1{}_3) \circ F(f_{28})(b) = F(g^1{}_3)(F(f_{28})(b)) = F(g^1{}_3)(b)) = g^2{}_3(b) = c$

- $F(g^1_3 o f_{28})(b) = F(g^1_8)(b) = c$ ومنه

وبالتالي يجب أن يكون $F(g^1_8) = g^2_8$

3) لأجل $F(g^2_3) = g^1_3$ حيث $g^2_3 o f_{28} = g^2_8$

- $F(g^2_3) o F(f_{28})(a) = F(g^2_3)(F(f_{28})(a)) =$

- $F(g^2_3)(a)) = g^1_3(a) = b$

- $F(g^2_3 o f_{28})(a) = F(g^2_8)(a) = b$ ومنه

- $F(g^2_3) o F(f_{28})(b) = F(g^2_3)(F(f_{28})(b)) = F(g^2_3)(b)) = g^1_3(b) = a$

$F(g^2_3 o f_{28})(b) = F(g^2_8)(b) = a$ ومنه

وبالتالي يجب أن يكون $F(g^2_8) = g^1_8$

4) لأجل $F(g^2_3) = g^2_3$

- $F(g^2_3) o F(f_{28})(a) = F(g^2_3)(F(f_{28})(a)) = F(g^2_3)(a)) = g^2_3(a) = b$

- ومنه $F(g^2_3 o f_{28})(a) = F(g^2_8)(a) = b$

$F(g^2_3) o F(f_{28})(b) = F(g^2_3)(F(f_{28})(b)) = F(g^2_3)(b)) = g^2_3(b) = c$

$F(g^2_3 o f_{28})(b) = F(g^2_8)(b) = c$ ومنه

وبالتالي يجب أن يكون $F(g^2_8) = g^2_8$

❖ **$g_4 o f_{28}: \quad G^1_{3,1} \longrightarrow G^1_{2,1} \longrightarrow G^1_{3,3}$**

❖ $f_{28}: G^1_{3,1} \longrightarrow G^1_{2,1}$ DO: $a \longrightarrow a$, $b \longrightarrow b$ DA: $(ab) \longrightarrow (ab)$

إن $g^1_4 o f_{28} = g^1_9$

1) لأجل $F(g^1_4) = g^1_4$

- $F(g^1_4) o F(f_{28})(a) = F(g^1_4)(F(f_{28})(a)) = F(g^1_4)(f_{28}(a) = g^1_4(a) = a$

- $F(g^1_4 o f_{28})(a) = F(g^1_9)(a) = a$ ومنه

- $F(g^1_4) \circ F(f_{28})(b) = F(g^1_4)(F(f_{28})(b)) = F(g^1_4)(f_{28}(b) = g^1_4(b) = b$

- $F(g^1_4 o f_{28})(b) = F(g^1_9)(b) = b$ ومنه

وبالتالي يجب أن يكون $F(g^1_9) = g^1_9$

2) لأجل $F(g^1_4) = g^2_4$

- $F(g^1_4) \circ F(f_{28})(a) = F(g^1_4)(F(f_{28})(a)) = F(g^1_4)(f_{28}(a) = g^2_4(a) = b$

- $F(g^1_4 o f_{28})(a) = F(g^1_9)(a) = b$ ومنه

- $F(g^1_4) \circ F(f_{28})(b) = F(g^1_4)(F(f_{28})(b)) = F(g^1_4)(f_{28}(b) = g^2_4(b) = c$

- $F(g^1_4 o f_{28})(b) = F(g^1_9)(b) = c$ ومنه

- وبالتالي يجب أن يكون $F(g^1_9) = g^2_9$

3) لأجل $F(g^1_4) = g^3_4$

- $F(g^1_4) \circ F(f_{28})(a) = F(g^1_4)(F(f_{28})(a)) = F(g^1_4)(f_{28}(a) = g^3_4(a) = c$

- $F(g^1_4 o f_{28})(a) = F(g^1_9)(a) = c$ ومنه

- $F(g^1_4) \circ F(f_{28})(b) = F(g^1_4)(F(f_{28})(b)) = F(g^1_4)(f_{28}(b) = g^3_4(b) = a$

- $F(g^1_4 o f_{28})(b) = F(g^1_9)(b) = a$

- وبالتالي يجب أن يكون $F(g^1_9) = g^3_9$

4) لأجل $F(g^2_4) = g^1_4$ حيث $g^2_4 o f_{28} = g^2_9$

- $F(g^2_4) \circ F(f_{28})(a) = F(g^2_4)(F(f_{28})(a)) = F(g^2_4)(f_{28}(a) = g^1_4(a) = a$

$F(g^2_4 o f_{28})(a) = F(g^2_9)(a) = a$ ومنه

$F(g^2_4) \circ F(f_{28})(b) = F(g^2_4)(F(f_{28})(b)) = F(g^2_4)(f_{28}(b) = g^1_4(b) = b$

$F(g^2_4 o f_{28})(b) = F(g^2_9)(b) = b$ ومنه

- وبالتالي يجب أن يكون $F(g^2_9) = g^1_9$

5) لأجل $F(g^2_4) = g^2_4$

- $F(g^2_4) \circ F(f_{28}) (a) = F (g^2_4) (F(f_{28}) (a)) = F(g^2_4)(f_{28} (a) = g^2_4 (a) =b$
- ومنه $F(g^2_4 of_{28}) (a) = F(g^2_9) (a) = b$
- $F(g^2_4) \circ F(f_{28}) (b) = F (g^2_4) (F(f_{28}) (b)) = F(g^2_4)(f_{28} (b) = g^2_4 (b) =c$
- ومنه $F(g^2_4 of_{28}) (b) = F(g^2_9) (b) = c$

وبالتالي يجب أن يكون $F(g^2_9) = g^2_9$

6) لأجل $F(g^2_4) = g^3_4$

- $F(g^2_4) \circ F(f_{28}) (a) = F (g^2_4) (F(f_{28}) (a)) = F(g^2_4)(f_{28} (a) = g^3_4 (a) =c$
- ومنه $F(g^2_4 of_{28}) (a) = F(g^{27}) (a) = c$
- $F(g^2_4) \circ F(f_{28}) (b) = F (g^2_4) (F(f_{28}) (b)) = F(g^2_4)(f_{28} (b) = g^3_4 (b) =a$
- ومنه $F(g^2_4 of_{28}) (b) = F(g^2_9) (b) = a$

وبالتالي يجب أن يكون $F(g^2_9) = g^3_9$

7) لأجل $F(g^3_4) = g^1_4$ إن $g^3_9 = g^3_4 of_{28}$

- $F(g^3_4) \circ F(f_{28}) (a) = F (g^3_4) (F(f_{28}) (a)) = F(g^3_4)(f_{28} (a) = g^1_4 (a) =a$
- ومنه $F(g^3_4 of_{28}) (a) = F(g^3_9) (a) = a$
- $F(g^3_4) \circ F(f_{28}) (b) = F (g^3_4) (F(f_{28}) (b)) = F(g^3_4)(f_{28} (b) = g^1_4 (b) =b$
- ومنه $F(g^3_4 of_{28}) (b) = F(g^3_9) (b) = b$

وبالتالي يجب أن يكون $F(g^3_9) = g^1_9$

8) لأجل $F(g^3_4) = g^2_4$

- $F(g^3_4) \circ F(f_{28}) (a) = F (g^3_4) (F(f_{28}) (a)) = F(g^3_4)(f_{28} (a) = g^2_4 (a) =b$
- ومنه $F(g^3_4 of_{28}) (a) = F(g^3_9) (a) = b$

- $F(g^3_4)\ o\ F(f_{28})\ (b) = F\ (g^3_4)\ (\ F(f_{28})\ (b)\ ) = F(g^3_4)(\ f_{28}\ (b) = g^2_4\ (b) = c$

- ومنه $F(g^3_4 o f_{28})\ (b) = F(\ g^3_9\ )\ (b) = c$

وبالتالي يجب أن يكون $F(g^3_9) = g^2_9$

9) لأجل $F(\ g^3_4) = g^3_4$

- $F(g^3_4)\ o\ F(f_{28})\ (a) = F\ (g^3_4)\ (\ F(f_{28})\ (a)\ ) = F(g^3_4)(\ f_{28}\ (a) = g^3_4\ (a) = c$

- ومنه $F(g^3_4 o f_{28})\ (a) = F(\ g^3_9\ )\ (a) = c$

- $F(g^3_4)\ o\ F(f_{28})\ (b) = F\ (g^3_4)\ (\ F(f_{28})\ (b)\ ) = F(g^3_4)(\ f_{28}\ (b) = g^3_4\ (b) = a$

ومنه $F(g^3_4 o f_{28})\ (b) = F(\ g^3_9\ )\ (b) = a$

وبالتالي يجب أن يكون $F(g^3_9) = g^3_9$

❖ $\mathbf{g_5 o f_{28}:\quad G^1_{3,1} \longrightarrow G^1_{2,1} \longrightarrow G^2_{3,3}}$

- $f_{28}: G^1_{3,1} \longrightarrow G^1_{2,1}$ DO:a $\longrightarrow$ a, b $\longrightarrow$ b DA:(ab) $\longrightarrow$ (ab)

إن $g^1_5 o f_{28} = g^1_{10}$

1) لأجل $F(\ g^1_5) = g^1_5$

- $F(g^1_5)\ o\ F(f_{28})\ (a) = F\ (g^1_5)\ (\ F(f_{28})\ (a)\ ) = F(g^1_5)(\ f_{28}\ (a) = g^1_5\ (a) = a$

- ومنه $F(g^1_5 o f_{28})\ (a) = F(\ g^1_{10}\ )\ (a) = a$

- $F(g^1_5)\ o\ F(f_{28})\ (b) = F\ (g^1_5)\ (\ F(f_{28})\ (b)\ ) = F(g^1_5)(\ f_{28}\ (b) = g^1_5\ (b) = b$

ومنه $F(g^1_5 o f_{28})\ (b) = F(\ g^1_{10}\ )\ (b) = b$

وبالتالي يجب أن يكون $F(g^1_{10}) = g^1_{10}$

2) لأجل $F(\ g^1_5) = g^2_5$

- $F(g^1_5) \circ F(f_{28})(a) = F(g^1_5)(F(f_{28})(a)) = F(g^1_5)(f_{28}(a) = g^2_5(a) = b$
- ومنه $F(g^1_5 o f_{28})(a) = F(g^1_{10})(a) = b$
- $F(g^1_5) \circ F(f_{28})(b) = F(g^1_5)(F(f_{28})(b)) = F(g^1_5)(f_{28}(b) = g^2_5(b) = c$
- ومنه $F(g^1_5 o f_{28})(b) = F(g^1_{10})(b) = c$
- وبالتالي يجب أن يكون $F(g^1_{10}) = g^2_{10}$

3) لأجل $F(g^1_5) = g^3_5$

- $F(g^1_5) \circ F(f_{28})(a) = F(g^1_5)(F(f_{28})(a)) = F(g^1_5)(f_{28}(a) = g^3_5(a) = a$
- ومنه $F(g^1_5 o f_{28})(a) = F(g^1_{10})(a) = a$
- $F(g^1_5) \circ F(f_{28})(b) = F(g^1_5)(F(f_{28})(b)) = F(g^1_5)(f_{28}(b) = g^3_5(b) = c$
- ومنه $F(g^1_5 o f_{28})(b) = F(g^1_{10})(b) = c$
- وبالتالي يجب أن يكون $F(g^1_{10}) = g^3_{10}$

4) لأجل $F(g^2_5) = g^1_5$ حيث $g^2_5 o f_{28} = g^2_{10}$

- $F(g^2_5) \circ F(f_{28})(a) = F(g^2_5)(F(f_{28})(a)) = F(g^2_5)(f_{28}(a) = g^1_5(a) = a$
- ومنه $F(g^2_5 o f_{28})(a) = F(g^2_{10})(a) = a$
- $F(g^2_5) \circ F(f_{28})(b) = F(g^2_5)(F(f_{28})(b)) = F(g^2_5)(f_{28}(b) = g^1_5(b) = b$
- ومنه $F(g^2_5 o f_{28})(b) = F(g^2_{10})(b) = b$
- وبالتالي يجب أن يكون $F(g^2_{10}) = g^1_{10}$

5) لأجل $F(g^2_5) = g^2_5$

- $F(g^2_5) \circ F(f_{28})(a) = F(g^2_5)(F(f_{28})(a)) = F(g^2_5)(f_{28}(a) = g^2_5(a) = b$
- ومنه $F(g^2_5 o f_{28})(a) = F(g^2_{10})(a) = b$
- $F(g^2_5) \circ F(f_{28})(b) = F(g^2_5)(F(f_{28})(b)) = F(g^2_5)(f_{28}(b) = g^2_5(b) = c$
- ومنه $F(g^2_5 o f_{28})(b) = F(g^2_{10})(b) = c$

$F(g^2_{10}) = g^2_{10}$ و بالتالي يجب أن يكون

6) لأجل $F(g^2_5) = g^3_5$

- $F(g^2_5)\ o\ F(f_{28})\ (a) = F\ (g^2_5)\ (\ F(f_{28})\ (a)\ ) = F(g^2_5)(\ f_{28}\ (a) = g^3_5\ (a) = a$
- ومنه $F(g^2_5 o f_{28})\ (a) = F(\ g^2_{10}\ )\ (a) = a$
- $F(g^2_5)\ o\ F(f_{28})\ (b) = F\ (g^2_5)\ (\ F(f_{28})\ (b)\ ) = F(g^2_5)(\ f_{28}\ (b) = g^3_5\ (b) = c$
- ومنه $F(g^2_5 o f_{28})\ (b) = F(\ g^2_{10}\ )\ (b) = c$

$F(g^2_{10}) = g^3_{10}$ و بالتالي يجب أن يكون

7) لأجل $F(g^3_5) = g^1_5$ إن $g^3_{10} = g^3_5 o f_{28}$

- $F(g^3_5)\ o\ F(f_{28})\ (a) = F\ (g^3_5)\ (\ F(f_{28})\ (a)\ ) = F(g^3_5)(\ f_{28}\ (a) = g^1_5\ (a) = a$
- ومنه $F(g^3_5 o f_{28})\ (a) = F(\ g^3_{10}\ )\ (a) = a$
- $F(g^3_5)\ o\ F(f_{28})\ (b) = F\ (g^3_5)\ (\ F(f_{28})\ (b)\ ) = F(g^3_5)(\ f_{28}\ (b) = g^1_5\ (b) = b$
- ومنه $F(g^3_5 o f_{28})\ (b) = F(\ g^3_{10}\ )\ (b) = b$

$F(g^3_{10}) = g^1_{10}$ وبالتالي يجب أن يكون

8) لأجل $F(g^3_5) = g^2_5$

- $F(g^3_5)\ o\ F(f_{28})\ (a) = F\ (g^3_5)\ (\ F(f_{28})\ (a)\ ) = F(g^3_5)(\ f_{28}\ (a) = g^2_5\ (a) = b$
- ومنه $F(g^3_5 o f_{28})\ (a) = F(\ g^3_{10}\ )\ (a) = b$
- $F(g^3_5)\ o\ F(f_{28})\ (b) = F\ (g^3_5)\ (\ F(f_{28})\ (b)\ ) = F(g^3_5)(\ f_{28}\ (b) = g^2_5\ (b) = c$
- ومنه $F(g^3_5 o f_{28})\ (b) = F(\ g^3_{10}\ )\ (b) = c$

$F(g^3_{10}) = g^2_{10}$ وبالتالي يجب أن يكون

8) لأجل $F(g^3_5) = g^3_5$

- $F(g^3_5) \circ F(f_{28})(a) = F(g^3_5)(F(f_{28})(a)) = F(g^3_5)(f_{28}(a) = g^3_5(a) = a$
- ومنه $F(g^3_5 o f_{28})(a) = F(g^3_{10})(a) = a$
- $F(g^3_5) \circ F(f_{28})(b) = F(g^3_5)(F(f_{28})(b)) = F(g^3_5)(f_{28}(b) = g^3_5(b) = c$
- ومنه $F(g^3_5 o f_{28})(b) = F(g^3_{10})(b) = c$

وبالتالي يجب أن يكون $F(g^3_{10}) = g^3_{10}$

❖ $g_6 o f_{27}: \quad G^1_{2,1} \longrightarrow G^1_{3,1} \longrightarrow G^1_{3,2}$

$f_{27}: G^1_{2,1} \longrightarrow G^1_{3,1}$ $\quad D_o: O_2 \longrightarrow O_3$ $\quad D_A: A^1_{2,1} \longrightarrow A^1_{3,1}$

- $a \longrightarrow a$ $\qquad (ab) \longrightarrow (ab)$

$b \longrightarrow b$

إن $g^1_6 o f_{27} = g^1_1$

1) لأجل $F(g^1_6) = g^1_6$

- $F(g^1_6) \circ F(f_{27})(a) = F(g^1_6)(F(f_{27})(a)) = F(g^1_6)((a) = g^1_6(a) = a$
- ومنه $F(g^1_6 o f_{27})(a) = F(g^1_1)(a) = a$

$F(g^1_6) \circ F(f_{27})(b) = F(g^1_6)(F(f_{27})(b)) = F(g^1_6)((b) = g^1_6(b) = b$

ومنه $F(g^1_6 o f_{27})(b) = F(g^1_1)(b) = b$

- وبالتالي يجب أن يكون $F(g^1_1) = g^1_1$

2) لأجل $F(g^1_6) = g^2_6$

- $F(g^1_6) \circ F(f_{27})(a) = F(g^1_6)(F(f_{27})(a)) = F(g^1_6)((a) = g^2_6(a) = b$

ومنه $F(g^1_6 o f_{27})(a) = F(g^1_1)(a) = b$

- $F(g^1_6) \circ F(f_{27})(b) = F(g^1_6)(F(f_{27})(b)) = F(g^1_6)((b) = g^2_6(b) = c$

ومنه $F(g^1_6 o f_{27})(b) = F(g^1_1)(b) = c$

- وبالتالي يجب أن يكون $F(g^1_1) = g^2_1$

3) لأجل $F(g^2_6) = g^1_6$ حيث $g^2_6 o f_{27} = g^2_1$

- $F(g^2_6)\ o\ F(f_{27})(a) = F(g^2_6)(F(f_{27})(a)) = F(g^2_6)((a) = g^1_6(a) = a$
- ومنه $F(g^2_6 o f_{27})(a) = F(g^2_1)(a) = a$
- $F(g^2_6)\ o\ F(f_{27})(b) = F(g^2_6)(F(f_{27})(b)) = F(g^2_6)((b) = g^1_6(b) = b$
- ومنه $F(g^2_6 o f_{27})(b) = F(g^2_1)(b) = b$
- وبالتالي يجب أن يكون $F(g^2_1) = g^1_1$

4) لأجل $F(g^2_6) = g^2_6$

- $F(g^2_6)\ o\ F(f_{27})(a) = F(g^2_6)(F(f_{27})(a)) = F(g^2_6)((a) = g^2_6(a) = b$
- ومنه $F(g^2_6 o f_{27})(a) = F(g^2_1)(a) = b$

$F(g^2_6)\ o\ F(f_{27})(b) = F(g^2_6)(F(f_{27})(b)) = F(g^2_6)((b) = g^2_6(b) = c$

ومنه $F(g^2_6 o f_{27})(b) = F(g^2_1)(b) = c$

- وبالتالي يجب أن يكون $F(g^2_1) = g^2_1$

❖ **$g_7 o f_{27}$: $G^1_{2,1} \longrightarrow G^1_{3,1} \longrightarrow G^2_{3,2}$**

- $f_{27}: G^1_{2,1} \longrightarrow G^1_{3,1}$ $D_A: A^1_{2,1} \longrightarrow A^1_{3,1}$ $D_o: O_2 \longrightarrow O_3$

$(ab) \longrightarrow (ab)$ $a \longrightarrow a$

$b \longrightarrow b$

إن $g^1_7 o f_{27} = g^1_2$

1) لأجل $F(g^1_7) = g^1_7$

- $F(g^1_7) \circ F(f_{27})(a) = F(g^1_7)(F(f_{27})(a)) = F(g^1_7)((a) = g^1_7(a) = a$
- ومنه $F(g^1_7 o f_{27})(a) = F(g^1_2)(a) = a$

$F(g^1_7) \circ F(f_{27})(b) = F(g^1_7)(F(f_{27})(b)) = F(g^1_7)((b) = g^1_7(b) = b$

ومنه $F(g^1_7 o f_{27})(b) = F(g^1_2)(b) = b$

- وبالتالي يجب أن يكون $F(g^1_2) = g^1_2$

2) لأجل $F(g^1_7) = g^2_7$

- $F(g^1_7) \circ F(f_{27})(a) = F(g^1_7)(F(f_{27})(a)) = F(g^1_7)((a) = g^2_7(a) = c$
- ومنه $F(g^1_7 o f_{27})(a) = F(g^1_2)(a) = c$
- $F(g^1_7) \circ F(f_{27})(b) = F(g^1_7)(F(f_{27})(b)) = F(g^1_7)((b) = g^2_7(b) = b$
- ومنه $F(g^1_7 o f_{27})(b) = F(g^1_2)(b) = b$
- وبالتالي يجب أن يكون $F(g^1_2) = g^2_2$

3) لأجل $F(g^2_7) = g^1_7$ حيث $g^2_7 o f_{27} = g^2_2$

$F(g^2_7) \circ F(f_{27})(a) = F(g^2_7)(F(f_{27})(a)) = F(g^2_7)((a) = g^1_7(a) = a$

- ومنه $F(g^2_7 o f_{27})(a) = F(g^2_2)(a) = a$
- $F(g^2_7) \circ F(f_{27})(b) = F(g^2_7)(F(f_{27})(b)) = F(g^2_7)((b) = g^1_7(b) = b$
- ومنه $F(g^2_7 o f_{27})(b) = F(g^2_2)(b) = b$
- وبالتالي يجب أن يكون $F(g^2_2) = g^1_2$

4) لأجل $F(g^2_7) = g^2_7$

- $F(g^2_7) \circ F(f_{27})(a) = F(g^2_7)(F(f_{27})(a)) = F(g^2_7)((a) = g^2_7(a) = c$
- ومنه $F(g^2_7 o f_{27})(a) = F(g^2_2)(a) = c$

$F(g^2_7)\ o\ F(f_{27})\ (b) = F\ (g^2_7)\ (\ F(f_{27})\ (b)\ ) = F(g^2_7)(\ (b) = g^2_7\ (b) = b$

ومنه $F(g^2_7of_{27})\ (b) = F(\ g^2_2\ )\ (b) = b$

- وبالتالي يجب أن يكون
- $F(g^2_2) = g^2_2$

❖ $g_8of_{27}:\quad G^1_{2,1} \longrightarrow G^1_{3,1} \longrightarrow G^3_{3,2}$

$f_{27}: G^1_{2,1} \longrightarrow G^1_{3,1}$ $D_o:\ O_2 \longrightarrow O_3$ $D_A: A^1_{2,1} \longrightarrow A^1_{3,1}$

- $(ab) \longrightarrow (ab)$ $a \longrightarrow a$

$b \longrightarrow b$

إن $g^1_8of_{27} = g^1_3$

1) لأجل $F(\ g^1_8) = g^1_8$

- $F(g^1_8)\ o\ F(f_{27})\ (a) = F\ (g^1_8)\ (\ F(f_{27})\ (a)\ ) = F(g^1_8)(\ (a) = g^1_8\ (a) = b$
- ومنه $F(g^1_8of_{27})\ (a) = F(\ g^1_3\ )\ (a) = b$

$F(g^1_8)\ o\ F(f_{27})\ (b) = F\ (g^1_8)\ (\ F(f_{27})\ (b)\ ) = F(g^1_8)(\ (b) = g^1_8\ (b) = a$

ومنه $F(g^1_8of_{27})\ (b) = F(\ g^1_3\ )\ (b) = a$

- وبالتالي يجب أن يكون $F(g^1_3) = g^1_3$

2) لأجل $F(\ g^1_8) = g^2_8$

- $F(g^1_8)\ o\ F(f_{27})\ (a) = F\ (g^1_8)\ (\ F(f_{27})\ (a)\ ) = F(g^1_8)(\ (a) = g^{26}\ (a) = b$

ومنه $F(g^1_8of_{27})\ (a) = F(\ g^1_3\ )\ (a) = b$

$F(g^1_8)\ o\ F(f_{27})\ (b) = F\ (g^1_8)\ (\ F(f_{27})\ (b)\ ) = F(g^1_8)(\ (b) = g^{26}\ (b) = c$

ومنه $F(g^1_8of_{27})\ (b) = F(\ g^1_3\ )\ (b) = c$

- وبالتالي يجب أن يكون $F(g^1_3) = g^2_3$

3) لأجل $F(g^2_8) = g^1_8$ حيث $g^2_8 o f_{27} = g^2_3$

- $F(g^{26}) \circ F(f_{27})(a) = F(g^{26})(F(f_{27})(a)) = F(g^{26})((a) = g^1_8(a) = b$

ومنه $F(g^{26} o f_{27})(a) = F(g^2_3)(a) = b$

- $F(g^{26}) \circ F(f_{27})(b) = F(g^{26})(F(f_{27})(b)) = F(g^{26})((b) = g^1_8(b) = a$

- ومنه $F(g^{26} o f_{27})(b) = F(g^2_3)(b) = a$

- وبالتالي يجب أن يكون $F(g^2_3) = g^1_3$

4) لأجل $F(g^2_8) = g^2_8$

- $F(g^{26}) \circ F(f_{27})(a) = F(g^{26})(F(f_{27})(a)) = F(g^{26})((a) = g^{26}(a) = b$

ومنه $F(g^{26} o f_{27})(a) = F(g^2_3)(a) = b$

$F(g^{26}) \circ F(f_{27})(b) = F(g^{26})(F(f_{27})(b)) = F(g^{26})((b) = g^{26}(b) = c$

ومنه $F(g^{26} o f_{27})(b) = F(g^2_3)(b) = c$

- وبالتالي يجب أن يكون $F(g^2_3) = g^2_3$

❖ $g_9 o f_{27}$: $G^1_{2,1} \longrightarrow G^1_{3,1} \longrightarrow G^1_{3,3}$

$f_{27}: G^1_{2,1} \longrightarrow G^1_{3,1}$ $D_o: O_2 \longrightarrow O_3$ $D_A: A^1_{2,1} \longrightarrow A^1_{3,1}$

- $(ab) \longrightarrow (ab)$ $a \longrightarrow a$

$b \longrightarrow b$

إن $g^1_9 o f_{27} = g^1_4$

1) لأجل $F(g^1_9) = g^1_9$

- $F(g^1_9) \circ F(f_{27})(a) = F(g^1_9)(F(f_{27})(a)) = F(g^1_9)((a) =$ $g^1_9(a) = a$

ومنه $F(g^1_9 o f_{27})(a) = F(g^1_4)(a) = a$

$F(g^1_9) o F(f_{27})(b) = F(g^1_9)(F(f_{27})(b)) = F(g^1_9)((b) = g^1_9(b) = b$

ومنه $F(g^1_9 o f_{27})(b) = F(g^1_4)(b) = b$

- وبالتالي يجب أن يكون $F(g^1_4) = g^1_4$

2) لأجل $F(g^1_9) = g^2_9$

$F(g^1_9) o F(f_{27})(a) = F(g^1_9)(F(f_{27})(a)) = F(g^1_9)((a) = g^{27}(a) = b$

ومنه $F(g^1_9 o f_{27})(a) = F(g^1_4)(a) = b$

- $F(g^1_9) o F(f_{27})(b) = F(g^1_9)(F(f_{27})(b)) = F(g^1_9)((b) = g^{27}(b) = c$

ومنه $F(g^1_9 o f_{27})(b) = F(g^1_4)(b) = c$

- وبالتالي يجب أن يكون $F(g^1_4) = g^2_4$

3) لأجل $F(g^1_9) = g^3_9$

- $F(g^1_9) o F(f_{27})(a) = F(g^1_9)(F(f_{27})(a)) = F(g^1_9)((a) = g^3_9(a) = c$

ومنه $F(g^1_9 o f_{27})(a) = F(g^1_4)(a) = c$

$F(g^1_9) o F(f_{27})(b) = F(g^1_9)(F(f_{27})(b)) = F(g^1_9)((b) = g^3_9(b) = a$

ومنه $F(g^1_9 o f_{27})(b) = F(g^1_4)(b) = a$

- وبالتالي يجب أن يكون $F(g^1_4) = g^3_4$

4) لأجل $F(g^2_9) = g^1_9$ حيث $g^2_9 o f_{27} = g^2_4$

- $F(g^{27}) o F(f_{27})(a) = F(g^{27})(F(f_{27})(a)) = F(g^{27})((a) = g^1_9(a) = a$

ومنه $F(g^{27} o f_{27})(a) = F(g^2_4)(a) = a$

- $F(g^{27}) o F(f_{27})(b) = F(g^{27})(F(f_{27})(b)) = F(g^{27})((b) = g^1_9(b) = b$

ومنه $F(g^{27} o f_{27})(b) = F(g^2_4)(b) = b$

- وبالتالي يجب أن يكون $F(g^2{}_4) = g^1{}_4$

5) لأجل $F(g^2{}_9) = g^2{}_9$

- $F(g^{27})\ o\ F(f_{27})\ (a) = F\ (g^{27})\ (\ F(f_{27})\ (a)\ ) = F(g^{27})(\ (a) = g^{27}\ (a) = b$

ومنه $F(g^{27} o f_{27})\ (a) = F(\ g^2{}_4\ )\ (a) = b$

$F(g^{27})\ o\ F(f_{27})\ (b) = F\ (g^{27})\ (\ F(f_{27})\ (b)\ ) = F(g^{27})(\ (b) = g^{27}\ (b) = c$

ومنه $F(g^{27} o f_{27})\ (b) = F(\ g^2{}_4\ )\ (b) = c$

وبالتالي يجب أن يكون $F(g^2{}_4) = g^2{}_4$

6) لأجل $F(g^2{}_9) = g^3{}_9$

- $F(g^{27})\ o\ F(f_{27})\ (a) = F\ (g^{27})\ (\ F(f_{27})\ (a)\ ) = F(g^{27})(\ (a) = g^3{}_9\ (a) = c$

ومنه $F(g^{27} o f_{27})\ (a) = F(\ g^2{}_4\ )\ (a) = c$

$F(g^{27})\ o\ F(f_{27})\ (b) = F\ (g^{27})\ (\ F(f_{27})\ (b)\ ) = F(g^{27})(\ (b) = g^3{}_9\ (b) = a$

ومنه $F(g^{27} o f_{27})\ (b) = F(\ g^2{}_4\ )\ (b) = a$

وبالتالي يجب أن يكون $F(g^2{}_4) = g^3{}_4$

7) لأجل $F(\ g^3{}_9) = g^1{}_9$ حيث $g^3{}_9 o f_{27} = g^3{}_4$

$F(g^3{}_9)\ o\ F(f_{27})\ (a) = F\ (g^3{}_9)\ (\ F(f_{27})\ (a)\ ) = F(g^3{}_9)(\ (a) = g^1{}_9\ (a) = a$

ومنه $F(g^3{}_9 o f_{27})\ (a) = F(\ g^3{}_4\ )\ (a) = a$

- $F(g^3{}_9)\ o\ F(f_{27})\ (b) = F\ (g^3{}_9)\ (\ F(f_{27})\ (b)\ ) = F(g^3{}_9)(\ (b) = g^1{}_9\ (b) = b$

ومنه $F(g^3{}_9 o f_{27})\ (b) = F(\ g^3{}_4\ )\ (b) = b$

- وبالتالي يجب أن يكون $F(g^3{}_4) = g^1{}_4$

8) لأجل $F(\ g^3{}_9) = g^2{}_9$

$F(g^3_9) \circ F(f_{27})\ (a) = F\ (g^3_9)\ (\ F(f_{27})\ (a)\ ) = F(g^3_9)(\ (a) = g^{27}\ (a) = b$

ومنه $F(g^3_9 o f_{27})\ (a) = F(\ g^3_4\ )\ (a) = b$

$F(g^3_9) \circ F(f_{27})\ (b) = F\ (g^3_9)\ (\ F(f_{27})\ (b)\ ) = F(g^3_9)(\ (b) = g^{27}\ (b) = c$

ومنه $F(g^3_9 o f_{27})\ (b) = F(\ g^3_4\ )\ (b) = c$

- وبالتالي يجب أن يكون $F(g^3_4) = g^2_4$

9) لأجل $F(\ g^3_9) = g^3_9$

- $F(g^3_9) \circ F(f_{27})\ (a) = F\ (g^3_9)\ (\ F(f_{27})\ (a)\ ) = F(g^3_9)(\ (a) = g^3_9\ (a) = c$

ومنه $F(g^3_9 o f_{27})\ (a) = F(\ g^3_4\ )\ (a) = c$

$F(g^3_9) \circ F(f_{27})\ (b) = F\ (g^3_9)\ (\ F(f_{27})\ (b)\ ) = F(g^3_9)(\ (b) = g^3_9\ (b) = a$

ومنه $F(g^3_9 o f_{27})\ (b) = F(\ g^3_4\ )\ (b) = a$

- وبالتالي يجب أن يكون $F(g^3_4) = g^3$

❖ $\mathbf{g_{10} o f_{27}:\quad G^1_{2,1} \longrightarrow G^1_{3,1} \longrightarrow G^2_{3,3}}$

$f_{27}: G^1_{2,1} \longrightarrow G^1_{3,1}\qquad D_o:\ O_2 \longrightarrow O_3\qquad D_A: A^1_{2,1} \longrightarrow A^1_{3,1}$

- $a \longrightarrow a\qquad (ab) \longrightarrow (ab)$

$b \longrightarrow b$

إن $g^1_{10} o f_{27} = g^1_5$

1) لأجل $F(\ g^1_{10}) = g^1_{10}$

- $F(g^1_{10}) \circ F(f_{27})\ (a) = F\ (g^1_{10})\ (\ F(f_{27})\ (a)\ ) = F(g^1_{10})(\ (a) = g^1_{10}(a) = a$

- ومنه $F(g^1_{10} o f_{27})\ (a) = F(\ g^1_5\ )\ (a) = a$

$F(g^1_{10}) \circ F(f_{27})\ (b) = F\ (g^1_{10})\ (\ F(f_{27})\ (b)\ ) = F(g^1_{10})(\ (b) = g^1_{10}\ (b) = b$

ومنه $F(g^1_{10} o f_{27})\ (b) = F(\ g^1_5\ )\ (b) = b$

- وبالتالي يجب أن يكون $F(g^1_5) = g^1_5$

2) لأجل $F(g^1_{10}) = g^2_{10}$

$F(g^1_{10})\ o\ F(f_{27}) (a) = F (g^1_{10}) (F(f_{27}) (a)) = F(g^1_{10})((a) = g^2_{10}(a) =b$

ومنه $F(g^1_{10} o f_{27}) (a) = F(g^1_5) (a) = b$

$F(g^1_{10})\ o\ F(f_{27}) (b) = F (g^1_{10}) (F(f_{27}) (b)) = F(g^1_{10})((b) = g^2_{10} (b) =c$

ومنه $F(g^1_{10} o f_{27}) (b) = F(g^1_5) (b) = c$

- ويالتالي يجب أن يكون $F(g^1_5) = g^2_5$

3) لأجل $F(g^1_{10}) = g^3_{10}$

- $F(g^1_{10})\ o\ F(f_{27}) (a) = F (g^1_{10}) (F(f_{27}) (a)) = F(g^1_{10})((a) = g^3_{10} (a) =a$

ومنه $F(g^1_{10} o f_{27}) (a) = F(g^1_5) (a) = a$

- $F(g^1_{10})\ o\ F(f_{27}) (b) = F (g^1_{10}) (F(f_{27}) (b)) = F(g^1_{10})((b) = g^3_{10} (b) =c$

ومنه $F(g^1_{10} o f_{27}) (b) = F(g^1_5) (b) = c$

- وبالتالي يجب أن يكون $F(g^1_5) = g^3_5$

4) لأجل $F(g^2_{10}) = g^1_{10}$ حيث $g^2_{10} o f_{27}= g^2_5$

- $F(g^2_{10})\ o\ F(f_{27}) (a) = F (g^2_{10}) (F(f_{27}) (a)) = F(g^2_{10})((a) = g^1_{10} (a) =a$

- ومنه $F(g^2_{10} o f_{27}) (a) = F(g^2_5) (a) = a$

- $F(g^2_{10})\ o\ F(f_{27}) (b) = F (g^2_{10}) (F(f_{27}) (b)) = F(g^2_{10})((b) = g^1_{10} (b) =b$

ومنه $F(g^2_{10} o f_{27}) (b) = F(g^2_5) (b) = b$

- وبالتالي يجب أن يكون $F(g^2_5) = g^1_5$

5) لأجل $F(g^2_{10}) = g^2_{10}$

$F(g^2_{10}) \circ F(f_{27})(a) = F(g^2_{10})(F(f_{27})(a)) = F(g^2_{10})((a) = g^2_{10}(a) = b$

- ومنه $F(g^2_{10} o f_{27})(a) = F(g^2_5)(a) = b$

$F(g^2_{10}) \circ F(f_{27})(b) = F(g^2_{10})(F(f_{27})(b)) = F(g^2_{10})((b) = g^2_{10}(b) = c$

ومنه $F(g^2_{10} o f_{27})(b) = F(g^2_5)(b) = c$

- وبالتالي يجب أن يكون $F(g^2_5) = g^2_5$

6) لأجل $F(g^2_{10}) = g^3_{10}$

- $F(g^2_{10}) \circ F(f_{27})(a) = F(g^2_{10})(F(f_{27})(a)) = F(g^2_{10})((a) = g^3_{10}(a) = a$

ومنه $F(g^2_{10} o f_{27})(a) = F(g^2_5)(a) = a$

$F(g^2_{10}) \circ F(f_{27})(b) = F(g^2_{10})(F(f_{27})(b)) = F(g^2_{10})((b) = g^3_{10}(b) = c$

ومنه $F(g^2_{10} o f_{27})(b) = F(g^2_5)(b) = c$

وبالتالي يجب أن يكون $F(g^2_5) = g^3_5$

7) لأجل $F(g^3_{10}) = g^1_{10}$ حيث $g^3_{10} o f_{27} = g^3_5$

$F(g^3_{10}) \circ F(f_{27})(a) = F(g^3_{10})(F(f_{27})(a)) = F(g^3_{10})((a) = g^1_{10}(a) = a$

ومنه $F(g^3_{10} o f_{27})(a) = F(g^3_5)(a) = a$

- $F(g^3_{10}) \circ F(f_{27})(b) = F(g^3_{10})(F(f_{27})(b)) = F(g^3_{10})((b) = g^1_{10}(b) = b$

ومنه $F(g^3_{10} o f_{27})(b) = F(g^3_5)(b) = b$

- وبالتالي يجب أن يكون $F(g^3_5) = g^1_5$

8) لأجل $F(g^3_{10}) = g^2_{10}$

$F(g^3_{10}) \circ F(f_{27})(a) = F(g^3_{10})(F(f_{27})(a)) = F(g^3_{10})((a) = g^2_{10}(a) = b$

ومنه $F(g^3_{10} o f_{27})(a) = F(g^3_5)(a) = b$

$F(g^3_{10}) \circ F(f_{27})(b) = F(g^3_{10})(F(f_{27})(b)) = F(g^3_{10})((b) = g^2_{10}(b) = c$

ومنه $F(g^3_{10} o f_{27})(b) = F(g^3_5)(b) = c$

- وبالتالي يجب أن يكون $F(g^3{}_5) = g^2{}_5$

9) لأجل $F(g^3{}_{10}) = g^3{}_{10}$

- $F(g^3{}_{10})$ o $F(f_{27})$ (a) = F $(g^3{}_{10})$ ($F(f_{27})$ (a)) = $F(g^3{}_{10})$((a) = $g^3{}_{10}$ (a) =a
- ومنه $F(g^3{}_{10}of_{27})$ (a) = F($g^3{}_5$) (a) = a

$F(g^3{}_{10})$ o $F(f_{27})$ (b) = F $(g^3{}_{10})$ ($F(f_{27})$ (b)) = $F(g^3{}_{10})$((b) = $g^3{}_{10}$ (b) =c

ومنه $F(g^3{}_{10}of_{27})$ (b) = F($g^3{}_5$) (b) = c

وبالتالي يجب أن يكون

$F(g^3{}_5) = g^3{}_5$

❖ **$f_{33}og_6$: $G^1{}_{3,1} \longrightarrow G^1{}_{3,2} \longrightarrow G^2{}_{3,3}$**

f_{33}: $G^1{}_{3,2} \longrightarrow G^2{}_{3,3}$

D_o: $O_3 \longrightarrow O_3$	D_A: $A^1{}_{3,2} \longrightarrow A^2{}_{3,3}$
a ⟶ a	(ab) ⟶ (ab)
b ⟶ b	(bc) ⟶ (bc)
c ⟶ c	

- إن $f_{33}og^1{}_6 = g^1{}_{10}$

1) لأجل $F(g^1{}_6) = g^1{}_6$

- $F(f_{33})$ o $F(g^1{}_6)$ (a) = F (f_{33}) ($F(g^1{}_6)$ (a)) = $F(f_{33})$($g^1{}_6$(a) = $F(f_{33})$ (a) =a
- ومنه $F(f_{33}og^1{}_6)$ (a) = F($g^1{}_{10}$) (a) = a
- $F(_{f33})$ o $F(^g{}_{16})$ (b) = F $(_{f33})$ ($F(^g{}_{16})$ (b)) = $F(_{f33})$($^g{}_{16}$(b) = $F(_{f33})$(b) =b

ومنه $F(f_{33}og^1{}_6)$ (b) = F($g^1{}_{10}$) (b) = b

- وبالتالي يجب أن يكون $F(g^1{}_{10}) = g^1{}_{10}$

2) لأجل $F(g^1_6) = g^2_6$

- $F(f_{33}) \circ F(g^1_6)(a) = F(f_{33})(F(g^1_6)(a)) = F(f_{33})(g^2_6(a)) = F(f_{33})(b) = b$

ومنه $F(f_{33} o g^1_6)(a) = F(g^1_{10})(a) = b$

- $F(f_{33}) \circ F(g^1_6)(b) = F(f_{33})(F(g^1_6)(b)) = F(f_{33})(g^2_6(b)) = F(f_{33})(c) = c$

ومنه $F(f_{33} o g^1_6)(b) = F(g^1_{10})(b) = c$

وبالتالي يجب أن يكون $F(g^1_{10}) = g^2_{10}$

3) لأجل $F(g^2_6) = g^1_6$ حيث $f_{33} o g^2_6 = g^2_{10}$

- $F(f_{33}) \circ F(g^2_6)(a) = F(f_{33})(F(g^2_6)(a)) = F(f_{33})(g^1_6(a)) = F(f_{33})(a) = a$

ومنه $F(f_{33} o g^2_6)(a) = F(g^2_{10})(a) = a$

- $F(f_{33}) \circ F(g^2_6)(b) = F(f_{33})(F(g^2_6)(b)) = F(f_{33})(g^1_6(b)) = F(f_{33})(b) = b$

ومنه $F(f_{33} o g^2_6)(b) = F(g^2_{10})(b) = b$

وبالتالي يجب أن يكون $F(g^2_{10}) = g^1_{10}$

4) لأجل $F(g^2_6) = g^2_6$

- $F(f_{33}) \circ F(g^2_6)(a) = F(f_{33})(F(g^2_6)(a)) = F(f_{33})(g^2_6(a)) = F(f_{33})(b) = b$

ومنه $F(f_{33} o g^2_6)(a) = F(g^2_{10})(a) = b$

- $F(f_{33}) \circ F(g^2_6)(b) = F(f_{33})(F(g^2_6)(b)) = F(f_{33})(g^2_6(b)) = F(f_{33})(c) = c$

ومنه $F(f_{33} o g^2_6)(b) = F(g^2_{10})(b) = c$

وبالتالي يجب أن يكون $F(g^2_{10}) = g^2_{10}$

❖ **$f_{41} o g_7$: $G^1_{3,1} \longrightarrow G^2_{3,2} \longrightarrow G^2_{3,3}$**

$f_{41}: G^2_{3,2} \longrightarrow G^2_{3,3}$ $D_o: O_3 \longrightarrow O_3$ $D_A: A^2_{3,2} \longrightarrow A^2_{3,3}$

- $(ab) \longrightarrow (bc)$ $\quad a \longrightarrow b$
  $(cb) \longrightarrow (ac)$ $\quad b \longrightarrow c$
- $c \longrightarrow a$
- إن $f_{41}og^1_7 = g^2_{10}$

1) لأجل $F(g^1_7) = g^1_7$

- $F(f_{41})\ o\ F(g^1_7)(a) = F(f_{41})(F(g^1_7)(a)) = F(f_{41})(g^1_7(a) = F(f_{41})(a) = b$
- ومنه $F(f_{41}og^1_7)(a) = F(g^2_{10})(a) = b$
- $F(f_{41})\ o\ F(g^1_7)(b) = F(f_{41})(F(g^1_7)(b)) = F(f_{41})(g^1_7(b) = F(f_{41})(b) = c$

ومنه $F(f_{41}og^1_7)(b) = F(g^2_{10})(b) = c$

- وبالتالي يجب أن يكون $F(g^2_{10}) = g^2_{10}$

2) لأجل $F(g^1_7) = g^2_7$

- $F(f_{41})\ o\ F(g^1_7)(a) = F(f_{41})(F(g^1_7)(a)) = F(f_{41})(g^2_7(a)) = F(f_{41})(c) = a$
- ومنه $F(f_{41}og^1_7)(a) = F(g^2_{10})(a) = a$
- $F(f_{41})\ o\ F(g^1_7)(b) = F(f_{41})(F(g^1_7)(b)) = F(f_{41})(g^2_7(b)) = F(f_{41})(b) = c$

ومنه $F(f_{41}og^1_7)(b) = F(g^2_{10})(b) = c$

وبالتالي يجب أن يكون $F(g^2_{10}) = g^3_{10}$

3) لأجل $F(g^2_7) = g^1_7$ حيث $f_{41}og^2_7 = g^3_{10}$

- $F(f_{41})\ o\ F(g^2_7)(a) = F(f_{41})(F(g^2_7)(a)) = F(f_{41})(g^1_7(a)) = F(f_{41})(a) = b$

ومنه $F(f_{41}og^2_7)(a) = F(g^3_{10})(a) = b$

- $F(f_{41})\ o\ F(g^2_7)(b) = F(f_{41})(F(g^2_7)(b)) = F(f_{41})(g^1_7(b)) = F(f_{41})(b) = c$

ومنه $F(f_{41}og^2_7)(b) = F(g^3_{10})(b) = c$

وبالتالي يجب أن يكون $F(g^3_{10}) = g^2_{10}$

4) لأجل $F(g^2_7) = g^2_7$

- $F(f_{41}) \circ F(g^2_7)(a) = F(f_{41})(F(g^2_7)(a)) = F(f_{41})(g^2_7(a)) = F(f_{41})(c) = a$
- ومنه $F(f_{41} o g^2_7)(a) = F(g^3_{10})(a) = a$
- $F(f_{41}) \circ F(g^2_7)(b) = F(f_{41})(F(g^2_7)(b)) = F(f_{41})(g^2_7(b)) = F(f_{41})(b) = c$

ومنه $F(f_{41} o g^2_7)(b) = F(g^3_{10})(b) = c$

وبالتالي يجب أن يكون $F(g^3_{10}) = g^3_{10}$

❖ **$f_{38} o g_8: \quad G^1_{3,1} \longrightarrow G^3_{3,2} \longrightarrow G^2_{3,3}$**

$f_{38}: G^3_{3,2} \longrightarrow G^2_{3,3}$ $\quad D_o: O_3 \longrightarrow O_3$ $\quad D_A: A^3_{3,2} \longrightarrow A^2_{3,3}$

- $(ba) \longrightarrow (ab)$ $\quad a \longrightarrow b$

$(bc) \longrightarrow (ac)$ $\quad b \longrightarrow a$

$c \longrightarrow c$

إن $f_{38} o g^1_8 = g^1_{10}$

1) لأجل $F(g^1_8) = g^1_8$

- $F(f_{38}) \circ F(g^1_8)(a) = F(f_{38})(F(g^1_8)(a)) = F(f_{38})(g^1_8(a) = F(f_{38})(b) = a$

ومنه $F(f_{38} o g^1_8)(a) = F(g^1_{10})(a) = a$

- $F(f_{38}) \circ F(g^1_8)(b) = F(f_{38})(F(g^1_8)(b)) = F(f_{38})(g^1_8(b) = F(f_{38})(a) = b$

ومنه $F(f_{38} o g^1_8)(b) = F(g^1_{10})(b) = b$

- وبالتالي يجب أن يكون $F(g^1_{10}) = g^1_{10}$

2) لأجل $F(g^1_8) = g^2_8$

- $F(f_{38}) \circ F(g^1_8)(a) = F(f_{38})(F(g^1_8)(a)) = F(f_{38})(g^{26}(a)) = F(f_{38})(b) = a$

ومنه $F(f_{38} o g^1_8)(a) = F(g^1_{10})(a) = a$

$F(f_{38}) \circ F(g^1_8)(b) = F(f_{38})(F(g^1_8)(b)) = F(f_{38})(g^{26}(b)) = F(f_{38})(c) = c$

ومنه $F(f_{38} o g^1_8)(b) = F(g^1_{10})(b) = c$

وبالتالي يجب أن يكون $F(g^1_{10}) = g^3_{10}$

3) لأجل $F(g_8^2) = g^1_8$ حيث $f_{38} o g^{26} = g^3_{10}$

- $F(f_{38}) \circ F(g^{26})(a) = F(f_{38})(F(g^{26})(a)) = F(f_{38})(g^1_8(a)) = F(f_{38})(b) = a$

ومنه $F(f_{38} o g^{26})(a) = F(g^3_{10})(a) = a$

- $F(f_{38}) \circ F(g^{26})(b) = F(f_{38})(F(g^{26})(b)) = F(f_{38})(g^1_8(b)) = F(f_{38})(a) = b$

ومنه $F(f_{38} o g^{26})(b) = F(g^3_{10})(b) = b$

وبالتالي يجب أن يكون $F(g^3_{10}) = g^1_{10}$

4) لأجل $F(g_8^2) = g_8^2$

- $F(f_{38}) \circ F(g^{26})(a) = F(f_{38})(F(g^{26})(a)) = F(f_{38})(g^{26}(a)) = F(f_{38})(b) = a$

ومنه $F(f_{38} o g^{26})(a) = F(g^3_{10})(a) = a$

$F(f_{38}) \circ F(g^{26})(b) = F(f_{38})(F(g^{26})(b)) = F(f_{38})(g^{26}(b)) = F(f_{38})(c) = c$

ومنه $F(f_{38} o g^{26})(b) = F(g^3_{10})(b) = c$

وبالتالي يجب أن يكون $F(g^3_{10}) = g^3_{10}$

❖ $g_{11} o g_1: \quad G^1_{2,1} \longrightarrow G^1_{3,2} \longrightarrow G^1_{3,3}$

1) لأجل $F(g^1_{11}) = g^1_{11}$ $\quad$ $F(g^1_1) = g^1_1$

إن $g^1_{11} o g^1_1 = g^1_4$

- $F(g^1_{11}) \circ F(g^1_1)(a) = F(g^1_{11})(F(g^1_1)(a)) = F(g^1_{11})(g^1_1(a)) = F(g^1_{11})(a) = a$

ومنه $F(g^1_{11} o g^1_1)(a) = F(g^1_4)(a) = a$

- $F(g^1_{11}) \circ F(g^1_1)(b) = F(g^1_{11})(F(g^1_1)(b)) = F(g^1_{11})(g^1_1(b)) = F(g^1_{11})(b) = b$

ومنه $F(g^1_{11} o g^1_1)(b) = F(g^1_4)(b) = b$

وبالتالي يجب أن يكون $F(g^1_4) = g^1_4$

2) لأجل $F(g^1_{11}) = g^2_{11}$ $\quad F(g^1_1) = g^1_1$

إن $g^1_{11} o g^1_1 = g^1_4$

- $F(g^1_{11}) \circ F(g^1_1)(a) = F(g^1_{11})(F(g^1_1)(a)) = F(g^1_{11})(g^1_1(a)) = F(g^1_{11})(a) = b$

ومنه $F(g^1_{11} o g^1_1)(a) = F(g^1_4)(a) = b$

$F(g^1_{11}) \circ F(g^1_1)(b) = F(g^1_{11})(F(g^1_1)(b)) = F(g^1_{11})(g^1_1(b)) = F(g^1_{11})(b) = c$

ومنه $F(g^1_{11} o g^1_1)(b) = F(g^1_4)(b) = c$

وبالتالي يجب أن يكون $F(g^1_4) = g^2_4$

3) لأجل إن $g^1_{11} o g^1_1 = g^1_4$

- $F(g^1_{11}) \circ F(g^1_1)(a) = F(g^1_{11})(F(g^1_1)(a)) = F(g^1_{11})(g^1_1(a)) = F(g^1_{11})(a) = c$

ومنه $F(g^1_{11} o g^1_1)(a) = F(g^1_4)(a) = c$

- $F(g^1_{11}) \circ F(g^1_1)(b) = F(g^1_{11})(F(g^1_1)(b)) = F(g^1_{11})(g^1_1(b)) = F(g^1_{11})(b) = a$

ومنه $F(g^1_{11} o g^1_1)(b) = F(g^1_4)(b) = a$

وبالتالي يجب أن يكون $F(g^1_4) = g^3_4$

4) لأجل $F(g^2_{11}) = g^2_{11}$ $\quad F(g^1_1) = g^1_1$

إن $g^2_{11} o g^1_1 = g^2_4$

- $F(g^2_{11}) \circ F(g^1_1)(a) = F(g^2_{11})(F(g^1_1)(a)) = F(g^2_{11})(g^1_1(a)) = F(g^2_{11})(a) = g^2_{11}(a) = b$

ومنه $F(g^{2}_{11} o g^{1}_{1})(a) = F(g^{2}_{4})(a) = b$

$F(g^{2}_{11}) o F(g^{1}_{1})(b) = F(g^{2}_{11})(F(g^{1}_{1})(b)) = F(g^{2}_{11})(g^{1}_{1}(b)) = F(g^{2}_{11})(b) = g^{2}_{11}(b) = c$

ومنه $F(g^{2}_{11} o g^{1}_{1})(b) = F(g^{2}_{4})(b) = c$

وبالتالي يجب أن يكون $F(g^{2}_{4}) = g^{2}_{4}$

5) لأجل $F(g^{2}_{11}) = g^{1}_{11}$ $\quad$ $F(g^{1}_{1}) = g^{1}_{1}$

إن $g^{2}_{11} o g^{1}_{1} = g^{2}_{4}$

- $F(g^{2}_{11}) o F(g^{1}_{1})(a) = F(g^{2}_{11})(F(g^{1}_{1})(a)) = F(g^{2}_{11})(g^{1}_{1}(a)) = F(g^{2}_{11})(a) = g^{1}_{11}(a) = a$

ومنه $F(g^{2}_{11} o g^{1}_{1})(a) = F(g^{2}_{4})(a) = a$

$F(g^{2}_{11}) o F(g^{1}_{1})(b) = F(g^{2}_{11})(F(g^{1}_{1})(b)) = F(g^{2}_{11})(g^{1}_{1}(b)) = F(g^{2}_{11})(b) = g^{1}_{11}(b) = b$

ومنه $F(g^{2}_{11} o g^{1}_{1})(b) = F(g^{2}_{4})(b) = b$

وبالتالي يجب أن يكون $F(g^{2}_{4}) = g^{1}_{4}$

6) لأجل $F(g^{2}_{11}) = g^{3}_{11}$ $\quad$ $F(g^{1}_{1}) = g^{1}_{1}$

إن $g^{2}_{11} o g^{1}_{1} = g^{2}_{4}$

- $F(g^{2}_{11}) o F(g^{1}_{1})(a) = F(g^{2}_{11})(F(g^{1}_{1})(a)) = F(g^{2}_{11})(g^{1}_{1}(a)) = F(g^{2}_{11})(a) = g^{3}_{11}(a) = c$

ومنه $F(g^{2}_{11} o g^{1}_{1})(a) = F(g^{2}_{4})(a) = c$

- $F(g^{2}_{11}) o F(g^{1}_{1})(b) = F(g^{2}_{11})(F(g^{1}_{1})(b)) = F(g^{2}_{11})(g^{1}_{1}(b)) = F(g^{2}_{11})(b) = g^{3}_{11}(b) = a$

ومنه $F(g^{2}_{11} o g^{1}_{1})(b) = F(g^{2}_{4})(b) = a$

وبالتالي يجب أن يكون $F(g^{2}_{4}) = g^{3}_{4}$

7) لأجل $F(g^{3}_{11}) = g^{3}_{11}$ $\quad$ $F(g^{1}_{1}) = g^{1}_{1}$

إن $g^{3}_{11} o g^{1}_{1} = g^{3}_{4}$

- $F(g^{3}_{11}) o F(g^{1}_{1})(a) = F(g^{3}_{11})(F(g^{1}_{1})(a)) = F(g^{3}_{11})(g^{1}_{1}(a)) = F(g^{3}_{11})(a) = g^{3}_{11}(a) = c$

ومنه $F(g^3_{11}og^1_1)\,(a) = F(\,g^3_4\,)\,(a) = c$

- $F(g^3_{11})\,o\,F(g^1_1)\,(b) = F\,(g^3_{11})\,(\,F(g^1_1)\,(b)\,) = F(g^3_{11})(\,g^1_1\,(b)\,) = F(g^3_{11})(b) = g^3_{11}(b) = a$

ومنه $F(g^3_{11}og^1_1)\,(b) = F(\,g^3_4\,)\,(b) = a$

وبالتالي يجب أن يكون $F(g^3_4) = g^3_4$

8) لأجل $F(\,g^3_{11}) = g^1_{11}$ $\quad$ $F(\,g^1_1) = g^1_1$

إن $g^3_{11}og^1_1 = g^3_4$

- $F(g^3_{11})\,o\,F(g^1_1\,)\,(a) = F\,(g^3_{11})\,(\,F(g^1_1)\,(a)\,) = F(g^3_{11})(\,g^1_1\,(a)) = F(g^3_{11})\,(a) = g^1_{11}\,(a) = a$

ومنه $F(g^3_{11}og^1_1)\,(a) = F(\,g^3_4\,)\,(a) = a$

- $F(g^3_{11})\,o\,F(g^1_1)\,(b) = F\,(g^3_{11})\,(\,F(g^1_1)\,(b)\,) = F(g^3_{11})(\,g^1_1\,(b)\,) = F(g^3_{11})(b) = g^1_{11}(b) = b$

ومنه $F(g^3_{11}og^1_1)\,(b) = F(\,g^3_4\,)\,(b) = b$

وبالتالي يجب أن يكون $F(g^3_4) = g^1_4$

9) لأجل $F(\,g^3_{11}) = g^2_{11}$ $\quad$ $F(\,g^1_1) = g^1_1$

إن $g^3_{11}og^1_1 = g^3_4$

- $F(g^3_{11})\,o\,F(g^1_1\,)\,(a) = F\,(g^3_{11})\,(\,F(g^1_1)\,(a)\,) = F(g^3_{11})(\,g^1_1\,(a)) = F(g^3_{11})\,(a) = g^2_{11}\,(a) = b$

ومنه $F(g^3_{11}og^1_1)\,(a) = F(\,g^3_4\,)\,(a) = b$

$F(g^3_{11})\,o\,F(g^1_1)\,(b) = F\,(g^3_{11})\,(\,F(g^1_1)\,(b)\,) = F(g^3_{11})(\,g^1_1\,(b)\,) = F(g^3_{11})(b) = g^2_{11}(b) = c$

ومنه $F(g^3_{11}og^1_1)\,(b) = F(\,g^3_4\,)\,(b) = c$

وباتالي يجب أن يكون $F(g^3_4) = g^2_4$

(10 لأجل $F(\,g^1_{11}) = g^1_{11}$ $\quad$ $F(\,g^1_1) = g^2_1$

إن $g^1_{11}og^1_1 = g^1_4$

- $F(g^1_{11})\,o\,F(g^1_1\,)\,(a) = F\,(g^1_{11})\,(\,F(g^1_1)\,(a)\,) = F(g^1_{11})(\,g^2_1\,(a)) = F(g^1_{11})\,(b) = g^1_{11}\,(b) = b$

ومنه $F(g^1_{11}og^1_1)\,(a) = F(\,g^1_4\,)\,(a) = b$

- $F(g^1_{11}) \circ F(g^1_1)(b) = F(g^1_{11})(F(g^1_1)(b)) = F(g^1_{11})(g^2_1(b)) = F(g^1_{11})(c) = g^1_{11}(c) = c$

ومنه $F(g^1_{11} o g^1_1)(b) = F(g^1_4)(b) = c$

وبالتالي يجب أن يكون $F(g^1_4) = g^2_4$

11) لأجل $F(g^1_{11}) = g^2_{11}$ $F(g^1_1) = g^2_1$

إن $g^1_{11} o g^1_1 = g^1_4$

- $F(g^1_{11}) \circ F(g^1_1)(a) = F(g^1_{11})(F(g^1_1)(a)) = F(g^1_{11})(g^2_1(a)) = F(g^1_{11})(b) = g^2_{11}(b) = c$

ومنه $F(g^1_{11} o g^1_1)(a) = F(g^1_4)(a) = c$

$F(g^1_{11}) \circ F(g^1_1)(b) = F(g^1_{11})(F(g^1_1)(b)) = F(g^1_{11})(g^2_1(b)) = F(g^1_{11})(c) = g^2_{11}(c) = a$

ومنه $F(g^1_{11} o g^1_1)(b) = F(g^1_4)(b) = a$

وبالتالي يجب أن يكون $F(g^1_4) = g^3_4$

12) لأجل $F(g^1_{11}) = g^3_{11}$ $F(g^1_1) = g^2_1$

إن $g^2_{11} o g^1_1 = g^2_4$

- $F(g^1_{11}) \circ F(g^1_1)(a) = F(g^1_{11})(F(g^1_1)(a)) = F(g^1_{11})(g^2_1(a)) = F(g^1_{11})(b) = g^3_{11}(b) = a$

ومنه $F(g^1_{11} o g^1_1)(a) = F(g^1_4)(a) = a$

- $F(g^1_{11}) \circ F(g^1_1)(b) = F(g^1_{11})(F(g^1_1)(b)) = F(g^1_{11})(g^2_1(b)) = F(g^1_{11})(c) = g^3_{11}(c) = b$

ومنه $F(g^1_{11} o g^1_1)(b) = F(g^1_4)(b) = b$

وبالتالي يجب أن يكون $F(g^1_4) = g^1_4$

13) لأجل $F(g^2_{11}) = g^2_{11}$ $F(g^1_1) = g^2_1$

إن $g^2_{11} o g^1_1 = g^2_4$

- $F(g^2_{11}) \circ F(g^1_1)(a) = F(g^2_{11})(F(g^1_1)(a)) = F(g^2_{11})(g^2_1(a)) = F(g^2_{11})(b) = g^2_{11}(b) = c$

ومنه $F(g^2_{11} o g^1_1)(a) = F(g^2_4)(a) = c$

- $F(g^2_{11}) \circ F(g^1_1)(b) = F(g^2_{11})(F(g^1_1)(b)) = F(g^2_{11})(g^2_1(b)) = F(g^2_{11})(c) = g^2_{11}(c) = b$

ومنه $F(g^2_{11}og^1_1)(b) = F(g^2_4)(b) = b$

وبالتالي يجب أن يكون $F(g^2_4) = g^3_4$

14) لأجل $F(g^2_{11}) = g^1_{11}$ ، $F(g^1_1) = g^2_1$

إن $g^2_{11}og^1_1 = g^2_4$

- $F(g^2_{11}) \circ F(g^1_1)(a) = F(g^2_{11})(F(g^1_1)(a)) = F(g^2_{11})(g^2_1(a)) = F(g^2_{11})(b) = g^1_{11}(b) = b$
- ومنه $F(g^2_{11}og^1_1)(a) = F(g^2_4)(a) = b$
- $F(g^2_{11}) \circ F(g^1_1)(b) = F(g^2_{11})(F(g^1_1)(b)) = F(g^2_{11})(g^2_1(b)) = F(g^2_{11})(c) = g^1_{11}(c) = c$

ومنه $F(g^2_{11}og^1_1)(b) = F(g^2_4)(b) = c$

وبالتالي يجب أن يكون $F(g^2_4) = g^2_4$

15) لأجل $F(g^2_{11}) = g^3_{11}$ ، $F(g^1_1) = g^2_1$

إن $g^2_{11}og^1_1 = g^2_4$

- $F(g^2_{11}) \circ F(g^1_1)(a) = F(g^2_{11})(F(g^1_1)(a)) = F(g^2_{11})(g^2_1(a)) = F(g^2_{11})(b) = g^3_{11}(b) = a$

ومنه $F(g^2_{11}og^1_1)(a) = F(g^2_4)(a) = a$

- $F(g^2_{11}) \circ F(g^1_1)(b) = F(g^2_{11})(F(g^1_1)(b)) = F(g^2_{11})(g^2_1(b)) = F(g^2_{11})(c) = g^3_{11}(c) = b$

ومنه $F(g^2_{11}og^1_1)(b) = F(g^2_4)(b) = b$

وبالتالي يجب أن يكون $F(g^2_4) = g^1_4$

16) لأجل $F(g^3_{11}) = g^3_{11}$ ، $F(g^1_1) = g^2_1$

إن $g^3_{11}og^1_1 = g^3_4$

- $F(g^3_{11}) \circ F(g^1_1)(a) = F(g^3_{11})(F(g^1_1)(a)) = F(g^3_{11})(g^2_1(a)) = F(g^3_{11})(b) = g^3_{11}(b) = a$

ومنه $F(g^3_{11}og^1_1)(a) = F(g^3_4)(a) = a$

- $F(g^3_{11}) \circ F(g^1_1)(b) = F(g^3_{11})(F(g^1_1)(b)) = F(g^3_{11})(g^2_1(b)) = F(g^3_{11})(c) = g^3_{11}(c) = b$

ومنه $F(g^3_{11} o g^1_1)(b) = F(g^3_4)(b) = b$

وبالتالي يجب أن يكون $F(g^3_4) = g^1_4$

17)لأجل $F(g^3_{11}) = g^1_{11}$ $F(g^1_1) = g^2_1$

إن $g^3_{11} o g^1_1 = g^3_4$

- $F(g^3_{11}) o F(g^1_1)(a) = F(g^3_{11})(F(g^1_1)(a)) = F(g^3_{11})(g^2_1(a)) = F(g^3_{11})(b) = g^1_{11}(b) = b$

ومنه $F(g^3_{11} o g^1_1)(a) = F(g^3_4)(a) = b$

- $F(g^3_{11}) o F(g^1_1)(b) = F(g^3_{11})(F(g^1_1)(b)) = F(g^3_{11})(g^2_1(b)) = F(g^3_{11})(c) = g^1_{11}(c) = c$

ومنه $F(g^3_{11} o g^1_1)(b) = F(g^3_4)(b) = c$

وبالتالي يجب أن يكون $F(g^3_4) = g^2_4$

18) لأجل $F(g^3_{11}) = g^2_{11}$ $F(g^1_1) = g^2_1$

إن $g^3_{11} o g^1_1 = g^3_4$

- $F(g^3_{11}) o F(g^1_1)(a) = F(g^3_{11})(F(g^1_1)(a)) = F(g^3_{11})(g^2_1(a)) = F(g^3_{11})(b) = g^2_{11}(b) = c$

ومنه $F(g^3_{11} o g^1_1)(a) = F(g^3_4)(a) = c$

- $F(g^3_{11}) o F(g^1_1)(b) = F(g^3_{11})(F(g^1_1)(b)) = F(g^3_{11})(g^2_1(b)) = F(g^3_{11})(c) = g^2_{11}(c) = a$

ومنه $F(g^3_{11} o g^1_1)(b) = F(g^3_4)(b) = a$

وبالتالي يجب أن يكون $F(g^3_4) = g^3_4$

19) لأجل $F(g^1_{11}) = g^1_{11}$ $F(g^2_1) = g^2_1$

إن $g^1_{11} o g^2_1 = g^2_4$

- $F(g^1_{11}) o F(g^2_1)(a) = F(g^1_{11})(F(g^2_1)(a)) = F(g^1_{11})(g^2_1(a)) = F(g^1_{11})(b) = g^1_{11}(b) = b$

ومنه $F(g^1_{11} o g^2_1)(a) = F(g^2_4)(a) = b$

- $F(g^1_{11}) o F(g^2_1)(b) = F(g^1_{11})(F(g^2_1)(b)) = F(g^1_{11})(g^2_1(b)) = F(g^1_{11})(c) = g^1_{11}(c) = c$

ومنه $F(g^1_{11} o g^2_1)(b) = F(g^2_4)(b) = c$

وبالتالي يجب أن يكون $F(g^2_4) = g^2_4$

20)لأجل $F(g^1_{11}) = g^2_{11}$ **$F(g^2_1) = g^2_1$**

إن $g^1_{11} o g^2_1 = g^2_4$

- $F(g^1_{11}) o F(g^2_1)(a) = F(g^1_{11})(F(g^2_1)(a)) = F(g^1_{11})(g^2_1(a)) = F(g^1_{11})(b) = g^2_{11}(b) = c$

ومنه $F(g^1_{11} o g^2_1)(a) = F(g^2_4)(a) = c$

$F(g^1_{11}) o F(g^2_1)(b) = F(g^1_{11})(F(g^2_1)(b)) = F(g^1_{11})(g^2_1(b)) = F(g^1_{11})(c) = g^2_{11}(c) = a$

ومنه $F(g^1_{11} o g^2_1)(b) = F(g^2_4)(b) = a$

ويالتالي يجب أن يكون $F(g^2_4) = g^3_4$

21) لأجل $F(g^1_{11}) = g^3_{11}$ **$F(g^2_1) = g^2_1$**

إن $g^1_{11} o g^2_1 = g^2_4$

- $F(g^1_{11}) o F(g^2_1)(a) = F(g^1_{11})(F(g^2_1)(a)) = F(g^1_{11})(g^2_1(a)) = F(g^1_{11})(b) = g^3_{11}(b) = a$

ومنه $F(g^1_{11} o g^2_1)(a) = F(g^2_4)(a) = a$

$F(g^1_{11}) o F(g^2_1)(b) = F(g^1_{11})(F(g^2_1)(b)) = F(g^1_{11})(g^2_1(b)) = F(g^1_{11})(c) = g^3_{11}(c) = b$

ومنه $F(g^1_{11} o g^2_1)(b) = F(g^2_4)(b) = b$

وبالتالي يجب أن يكون $F(g^2_4) = g^1_4$

22) لأجل $F(g^2_{11}) = g^2_{11}$ **$F(g^2_1) = g^2_1$**

إن $g^2_{11} o g^2_1 = g^3_4$

- $F(g^2_{11}) o F(g^2_1)(a) = F(g^2_{11})(F(g^2_1)(a)) = F(g^2_{11})(g^2_1(a)) = F(g^2_{11})(b) = g^2_{11}(b) = c$

ومنه $F(g^2_{11} o g^2_1)(a) = F(g^3_4)(a) = c$

- $F(g^2_{11}) o F(g^2_1)(b) = F(g^2_{11})(F(g^2_1)(b)) = F(g^2_{11})(g^2_1(b)) = F(g^2_{11})(c) = g^2_{11}(c) = a$

ومنه $F(g^2_{11} o g^2_1)(b) = F(g^3_4)(b) = a$

وبالتالي يجب أن يكون $F(g^3_4) = g^3_4$

23) لأجل $F(g^{2}_{11}) = g^{1}_{11}$ $\quad$ $F(g^{2}_{1}) = g^{2}_{1}$

إن $g^{2}_{11} o g^{2}_{1} = g^{3}_{4}$

- $F(g^{2}_{11}) \circ F(g^{2}_{1})(a) = F(g^{2}_{11})(F(g^{2}_{1})(a)) = F(g^{2}_{11})(g^{2}_{1}(a)) = F(g^{2}_{11})(b) = g^{1}_{11}(b) = b$

ومنه $F(g^{2}_{11} o g^{2}_{1})(a) = F(g^{3}_{4})(a) = b$

$F(g^{2}_{11}) \circ F(g^{2}_{1})(b) = F(g^{2}_{11})(F(g^{2}_{1})(b)) = F(g^{2}_{11})(g^{2}_{1}(b)) = F(g^{2}_{11})(c) = g^{1}_{11}(c) = c$

ومنه $F(g^{2}_{11} o g^{2}_{1})(b) = F(g^{3}_{4})(b) = c$

وبالتالي يجب أن يكون $F(g^{3}_{4}) = g^{2}_{4}$

24) لأجل $F(g^{2}_{11}) = g^{3}_{11}$ $\quad$ $F(g^{2}_{1}) = g^{2}_{1}$

إن $g^{2}_{11} o g^{2}_{1} = g^{3}_{4}$

- $F(g^{2}_{11}) \circ F(g^{2}_{1})(a) = F(g^{2}_{11})(F(g^{2}_{1})(a)) = F(g^{2}_{11})(g^{2}_{1}(a)) = F(g^{2}_{11})(b) = g^{3}_{11}(b) = a$

ومنه $F(g^{2}_{11} o g^{2}_{1})(a) = F(g^{3}_{4})(a) = a$

$F(g^{2}_{11}) \circ F(g^{2}_{1})(b) = F(g^{2}_{11})(F(g^{2}_{1})(b)) = F(g^{2}_{11})(g^{2}_{1}(b)) = F(g^{2}_{11})(c) = g^{3}_{11}(c) = b$

ومنه $F(g^{2}_{11} o g^{2}_{1})(b) = F(g^{3}_{4})(b) = b$

وبالتالي يجب أن يكون $F(g^{3}_{4}) = g^{1}_{4}$

25) لأجل $F(g^{3}_{11}) = g^{3}_{11}$ $\quad$ $F(g^{2}_{1}) = g^{2}_{1}$

إن $g^{3}_{11} o g^{2}_{1} = g^{1}_{4}$

- $F(g^{3}_{11}) \circ F(g^{2}_{1})(a) = F(g^{3}_{11})(F(g^{2}_{1})(a)) = F(g^{3}_{11})(g^{2}_{1}(a)) = F(g^{3}_{11})(b) = g^{3}_{11}(b) = a$

ومنه $F(g^{3}_{11} o g^{2}_{1})(a) = F(g^{1}_{4})(a) = a$

$F(g^{3}_{11}) \circ F(g^{2}_{1})(b) = F(g^{3}_{11})(F(g^{2}_{1})(b)) = F(g^{3}_{11})(g^{2}_{1}(b)) = F(g^{3}_{11})(c) = g^{3}_{11}(c) = b$

ومنه $F(g^{3}_{11} o g^{2}_{1})(b) = F(g^{1}_{4})(b) = b$

وبالتالي يجب أن يكون $F(g^{1}_{4}) = g^{1}_{4}$

26) لأجل $F(g^{3}_{11}) = g^{1}_{11}$ **$F(g^{2}_{1}) = g^{2}_{1}$**

إن $g^{3}_{11} o g^{2}_{1} = g^{1}_{4}$

- $F(g^{3}_{11})\ o\ F(g^{2}_{1})(a) = F(g^{3}_{11})(F(g^{2}_{1})(a)) = F(g^{3}_{11})(g^{2}_{1}(a)) = F(g^{3}_{11})(b) = g^{1}_{11}(b) = b$

ومنه $F(g^{3}_{11} o g^{2}_{1})(a) = F(g^{1}_{4})(a) = b$

$F(g^{3}_{11})\ o\ F(g^{2}_{1})(b) = F(g^{3}_{11})(F(g^{2}_{1})(b)) = F(g^{3}_{11})(g^{2}_{1}(b)) = F(g^{3}_{11})(c) = g^{1}_{11}(c) = c$

ومنه $F(g^{3}_{11} o g^{2}_{1})(b) = F(g^{1}_{4})(b) = c$

وبالتالي يجب أن يكون $F(g^{1}_{4}) = g^{2}_{4}$

27) لأجل $F(g^{3}_{11}) = g^{2}_{11}$ **$F(g^{2}_{1}) = g^{2}_{1}$**

إن $g^{3}_{11} o g^{2}_{1} = g^{1}_{4}$

- $F(g^{3}_{11})\ o\ F(g^{2}_{1})(a) = F(g^{3}_{11})(F(g^{2}_{1})(a)) = F(g^{3}_{11})(g^{2}_{1}(a)) = F(g^{3}_{11})(b) = g^{2}_{11}(b) = c$

ومنه $F(g^{3}_{11} o g^{2}_{1})(a) = F(g^{1}_{4})(a) = c$

$F(g^{3}_{11})\ o\ F(g^{2}_{1})(b) = F(g^{3}_{11})(F(g^{2}_{1})(b)) = F(g^{3}_{11})(g^{2}_{1}(b)) = F(g^{3}_{11})(c) = g^{2}_{11}(c) = a$

ومنه $F(g^{3}_{11} o g^{2}_{1})(b) = F(g^{1}_{4})(b) = a$

وبالتالي يجب أن يكون $F(g^{1}_{4}) = g^{3}_{4}$

28) لأجل $F(g^{1}_{11}) = g^{1}_{11}$ **$F(g^{2}_{1}) = g^{1}_{1}$**

إن $g^{1}_{11} o g^{2}_{1} = g^{2}_{4}$

- $F(g^{1}_{11})\ o\ F(g^{2}_{1})(a) = F(g^{1}_{11})(F(g^{2}_{1})(a)) = F(g^{1}_{11})(g^{1}_{1}(a)) = F(g^{1}_{11})(a) = g^{1}_{11}(a) = a$

ومنه $F(g^{1}_{11} o g^{2}_{1})(a) = F(g^{2}_{4})(a) = a$

- $F(g^{1}_{11})\ o\ F(g^{2}_{1})(b) = F(g^{1}_{11})(F(g^{2}_{1})(b)) = F(g^{1}_{11})(g^{1}_{1}(b)) = F(g^{1}_{11})(b) = g^{1}_{11}(b) = b$

ومنه $F(g^{1}_{11} o g^{2}_{1})(b) = F(g^{2}_{4})(b) = b$

و بالتالي يجب أن يكون $F(g^{2}_{4}) = g^{1}_{4}$

29) لأجل $F(g^1_{11}) = g^2_{11}$ **$F(g^2_1) = g^1_1$**

إن $g^1_{11} o g^2_1 = g^2_4$

$F(g^1_{11})\, o\, F(g^2_1)(a) = F(g^1_{11})(F(g^2_1)(a)) = F(g^1_{11})(g^1_1(a)) = F(g^1_{11})(a) = g^2_{11}(a) = b$

- ومنه $F(g^1_{11} o g^2_1)(a) = F(g^2_4)(a) = b$

- $F(g^1_{11})\, o\, F(g^2_1)(b) = F(g^1_{11})(F(g^2_1)(b)) = F(g^1_{11})(g^1_1(b)) = F(g^1_{11})(b) = g^2_{11}(b) = c$

ومنه $F(g^1_{11} o g^2_1)(b) = F(g^2_4)(b) = c$

وبالتالي يجب أن يكون $F(g^2_4) = g^2_4$

30) لأجل $F(g^1_{11}) = g^3_{11}$ **$F(g^2_1) = g^1_1$**

إن $g^1_{11} o g^2_1 = g^2_4$

- $F(g^1_{11})\, o\, F(g^2_1)(a) = F(g^1_{11})(F(g^2_1)(a)) = F(g^1_{11})(g^1_1(a)) = F(g^1_{11})(a) = g^3_{11}(a) = c$

ومنه $F(g^1_{11} o g^2_1)(a) = F(g^2_4)(a) = c$

- $F(g^1_{11})\, o\, F(g^2_1)(b) = F(g^1_{11})(F(g^2_1)(b)) = F(g^1_{11})(g^1_1(b)) = F(g^1_{11})(b) = g^3_{11}(b) = a$

- ومنه $F(g^1_{11} o g^2_1)(b) = F(g^2_4)(b) = a$

و بالتالي يجب أن يكون $F(g^2_4) = g^3_4$

31) لأجل $F(g^2_{11}) = g^2_{11}$ **$F(g^2_1) = g^1_1$**

إن $g^2_{11} o g^2_1 = g^3_4$

- $F(g^2_{11})\, o\, F(g^2_1)(a) = F(g^2_{11})(F(g^2_1)(a)) = F(g^2_{11})(g^1_1(a)) = F(g^2_{11})(a) = g^2_{11}(a) = b$

ومنه $F(g^2_{11} o g^2_1)(a) = F(g^3_4)(a) = b$

$F(g^2_{11})\, o\, F(g^2_1)(b) = F(g^2_{11})(F(g^2_1)(b)) = F(g^2_{11})(g^1_1(b)) = F(g^2_{11})(b) = g^2_{11}(b) = c$

ومنه $F(g^2_{11} o g^2_1)(b) = F(g^3_4)(b) = c$

و بالتالي يجب أن يكون $F(g^3_4) = g^2_4$

$F(g^2_1) = g^1_1$ لأجل $F(g^2_{11}) = g^1_{11}$ (32

إن $g^2_{11} o g^2_1 = g^3_4$

- $F(g^2_{11})$ o $F(g^2_1)$ (a) = F (g^2_{11}) ($F(g^2_1)$ (a)) = $F(g^2_{11})$(g^1_1 (a))= $F(g^2_{11})$ (a) = g^1_{11} (a) =a

- ومنه $F(g^2_{11} o g^2_1)$ (a) = F(g^3_4) (a) = a

- $F(g^2_{11})$ o $F(g^2_1)$ (b) = F (g^2_{11}) ($F(g^2_1)$ (b)) = $F(g^2_{11})$(g^1_1 (b))= $F(g^2_{11})$(b) = g^1_{11}(b) =b
ومنه $F(g^2_{11} o g^2_1)$ (b) = F(g^3_4) (b) = b

وبالتالي يجب أن يكون $F(g^3_4) = g^1_4$

$F(g^2_1) = g^1_1$ لأجل $F(g^2_{11}) = g^3_{11}$ (33

إن $g^2_{11} o g^2_1 = g^3_4$

- $F(g^2_{11})$ o $F(g^2_1)$ (a) = F (g^2_{11}) ($F(g^2_1)$ (a)) = $F(g^2_{11})$(g^1_1 (a))= $F(g^2_{11})$ (a) = g^3_{11} (a) =c

ومنه $F(g^2_{11} o g^2_1)$ (a) = F(g^3_4) (a) = c

- $F(g^2_{11})$ o $F(g^2_1)$ (b) = F (g^2_{11}) ($F(g^2_1)$ (b)) = $F(g^2_{11})$(g^1_1 (b))= $F(g^2_{11})$(b) = g^3_{11}(b) =a

ومنه $F(g^2_{11} o g^2_1)$ (b) = F(g^3_4) (b) = a

وبالتالي يجب أن يكون $F(g^3_4) = g^3_4$

$F(g^2_1) = g^1_1$ لأجل $F(g^3_{11}) = g^3_{11}$ (34

إن $g^3_{11} o g^2_1 = g^1_4$

- $F(g^3_{11})$ o $F(g^2_1)$ (a) = F (g^3_{11}) ($F(g^2_1)$ (a)) = $F(g^3_{11})$(g^1_1 (a))= $F(g^3_{11})$ (a) = g^3_{11} (a) =c

ومنه $F(g^3_{11} o g^2_1)$ (a) = F(g^1_4) (a) = c

- $F(g^3_{11})$ o $F(g^2_1)$ (b) = F (g^3_{11}) ($F(g^2_1)$ (b)) = $F(g^3_{11})$(g^1_1 (b))= $F(g^3_{11})$(b) = g^3_{11}(b) =a
ومنه $F(g^3_{11} o g^2_1)$ (b) = F(g^1_4) (b) = a

وبالتالي يجب أن يكون $F(g^1_4) = g^3_4$

$F(g^2_1) = g^1_1$ لأجل $F(g^3_{11}) = g^1_{11}$ (35

إن $g^3_{11}og^2_1= g^1_4$

- $F(g^3_{11})\ o\ F(g^2_1)(a) = F(g^3_{11})(F(g^2_1)(a)) = F(g^3_{11})(g^1_1(a))= F(g^3_{11})(a) = g^1_{11}(a) =a$

ومنه $F(g^3_{11}og^2_1)(a) = F(g^1_4)(a) = a$

- $F(g^3_{11})\ o\ F(g^2_1)(b) = F(g^3_{11})(F(g^2_1)(b)) = F(g^3_{11})(g^1_1(b))= F(g^3_{11})(b) = g^1_{11}(b) =b$

ومنه $F(g^3_{11}og^2_1)(b) = F(g^1_4)(b) = b$

وبالتالي يجب أن يكون $F(g^1_4) = g^1_4$

36) لأجل $F(g^3_{11}) = g^2_{11}$ $\quad F(g^2_1) = g^1_1$

إن $g^3_{11}og^2_1= g^1_4$

- $F(g^3_{11})\ o\ F(g^2_1)(a) = F(g^3_{11})(F(g^2_1)(a)) = F(g^3_{11})(g^1_1(a))= F(g^3_{11})(a) = g^2_{11}(a) =b$

ومنه $F(g^3_{11}og^2_1)(a) = F(g^1_4)(a) = b$

- $F(g^3_{11})\ o\ F(g^2_1)(b) = F(g^3_{11})(F(g^2_1)(b)) = F(g^3_{11})(g^1_1(b))= F(g^3_{11})(b) = g^2_{11}(b) =c$

ومنه $F(g^3_{11}og^2_1)(b) = F(g^1_4)(b) = c$

وبالتالي يجب أن يكون $F(g^1_4) = g^2$

❖ $g_{11}og_6: \quad G^1_{3,1} \longrightarrow G^1_{3,2} \longrightarrow G^1_{3,3}$

1) لأجل $F(g^1_{11}) = g^1_{11}$ $\quad F(g^1_6) = g^1_6$

إن $g^1_{11}og^1_1= g^1_9$

- $F(g^1_{11})\ o\ F(g^1_6)(a) = F(g^1_{11})(F(g^1_6)(a)) = F(g^1_{11})(g^1_6(a))= F(g^1_{11})(a) =a$

ومنه $F(g^1_{11}og^1_6)(a) = F(g^1_9)(a) = a$

- $F(g^1_{11})\ o\ F(g^1_6)(b) = F(g^1_{11})(F(g^1_6)(b)) = F(g^1_{11})(g^1_6(b))= F(g^1_{11})(b) =b$

ومنه $F(g^1_{11}og^1_6)(b) = F(g^1_9)(b) = b$

وبالتالي يجب أن يكون $F(g^1_9) = g^1_9$

2) لأجل $F(g^1_{11}) = g^2_{11}$ **$F(g^1_6) = g^1_6$**

إن $g^1_{11} o g^1_6 = g^1_9$

- $F(g^1_{11}) o F(g^1_6)(a) = F(g^1_{11})(F(g^1_6)(a)) = F(g^1_{11})(g^1_6(a)) = F(g^1_{11})(a) = b$

ومنه $F(g^1_{11} o g^1_6)(a) = F(g^1_9)(a) = b$

- $F(g^1_{11}) o F(g^1_6)(b) = F(g^1_{11})(F(g^1_6)(b)) = F(g^1_{11})(g^1_6(b)) = F(g^1_{11})(b) = c$

ومنه $F(g^1_{11} o g^1_6)(b) = F(g^1_9)(b) = c$

و بالتالي يجب أن يكون $F(g^1_9) = g^2_9$

3) لأجل $F(g^1_{11}) = g^3_{11}$ **$F(g^1_6) = g^1_6$**

إن $g^1_{11} o g^1_6 = g^1_9$

- $F(g^1_{11}) o F(g^1_6)(a) = F(g^1_{11})(F(g^1_6)(a)) = F(g^1_{11})(g^1_6(a)) = F(g^1_{11})(a) = c$

ومنه $F(g^1_{11} o g^1_6)(a) = F(g^1_9)(a) = c$

$F(g^1_{11}) o F(g^1_6)(b) = F(g^1_{11})(F(g^1_6)(b)) = F(g^1_{11})(g^1_6(b)) = F(g^1_{11})(b) = a$

ومنه $F(g^1_{11} o g^1_6)(b) = F(g^1_9)(b) = a$

و بالتالي يجب أن يكون $F(g^1_9) = g^3_9$

4) لأجل $F(g^2_{11}) = g^2_{11}$ **$F(g^1_6) = g^1_6$**

إن $g^2_{11} o g^1_6 = g^2_9$

- $F(g^2_{11}) o F(g^1_6)(a) = F(g^2_{11})(F(g^1_6)(a)) = F(g^2_{11})(g^1_6(a)) = F(g^2_{11})(a) = g^2_{11}(a) = b$

ومنه $F(g^2_{11} o g^1_6)(a) = F(g^2_9)(a) = b$

- $F(g^2_{11}) o F(g^1_6)(b) = F(g^2_{11})(F(g^1_6)(b)) = F(g^2_{11})(g^1_6(b)) = F(g^2_{11})(b) = g^2_{11}(b) = c$

ومنه $F(g^2_{11} o g^1_6)(b) = F(g^2_9)(b) = c$

و بالتالي يجب أن يكون $F(g^2_9) = g^2_9$

$F(g^2_{11}) = g^1_{11}$ لأجل (5 **$F(g^1_6) = g^1_6$**

إن $g^2_{11} o g^1_6 = g^2_9$

- $F(g^2_{11}) \, o \, F(g^1_6)\,(a) = F(g^2_{11})\,(F(g^1_6)\,(a)) = F(g^2_{11})(g^1_6\,(a)) = F(g^2_{11})\,(a) = g^1_{11}\,(a) = a$

ومنه $F(g^2_{11} o g^1_6)\,(a) = F(g^2_9)\,(a) = a$

$F(g^2_{11}) \, o \, F(g^1_6)\,(b) = F(g^2_{11})\,(F(g^1_6)\,(b)) = F(g^2_{11})(g^1_6\,(b)) = F(g^2_{11})(b) = g^1_{11}(b) = b$

ومنه $F(g^2_{11} o g^1_6)\,(b) = F(g^2_9)\,(b) = b$

و بالتالي يجب أن يكون $F(g^2_9) = g^1_9$

$F(g^2_{11}) = g^3_{11}$ لأجل (6 **$F(g^1_6) = g^1_6$**

إن $g^2_{11} o g^1_6 = g^2_9$

- $F(g^2_{11}) \, o \, F(g^1_6)\,(a) = F(g^2_{11})\,(F(g^1_6)\,(a)) = F(g^2_{11})(g^1_6(a)) = F(g^2_{11})\,(a) = g^3_{11}\,(a) = c$

ومنه $F(g^2_{11} o g^1_6)\,(a) = F(g^2_9)\,(a) = c$

- $F(g^2_{11}) \, o \, F(g^1_6)\,(b) = F(g^2_{11})\,(F(g^1_6)\,(b)) = F(g^2_{11})(g^1_6\,(b)) = F(g^2_{11})(b) = g^3_{11}(b) = a$

ومنه $F(g^2_{11} o g^1_6)\,(b) = F(g^2_9)\,(b) = a$

و بالتالي يجب أن يكون $F(g^2_9) = g^3_9$

$F(g^3_{11}) = g^3_{11}$ لأجل (7 **$F(g^1_6) = g^1_6$**

إن $g^3_{11} o g^1_6 = g^3_9$

- $F(g^3_{11}) \, o \, F(g^1_6)\,(a) = F(g^3_{11})\,(F(g^1_6)\,(a)) = F(g^3_{11})(g^1_6(a)) = F(g^3_{11})\,(a) = g^3_{11}\,(a) = c$

ومنه $F(g^3_{11} o g^1_6)\,(a) = F(g^3_9)\,(a) = c$

$F(g^3_{11}) \, o \, F(g^1_6)\,(b) = F(g^3_{11})\,(F(g^1_6)\,(b)) = F(g^3_{11})(g^1_6\,(b)) = F(g^3_{11})(b) = g^3_{11}(b) = a$

ومنه $F(g^3_{11} o g^1_6)\,(b) = F(g^3_9)\,(b) = a$

و بالتالي يجب أن يكون $F(g^3_9) = g^3_9$

$F(g^3_{11}) = g^1_{11}$ لأجل (8 **$F(g^1_6) = g^1_6$**

إن $g^3_{11} o g^1_6 = g^3_9$

- $F(g^3_{11}) \, o \, F(g^1_6)(a) = F(g^3_{11})(F(g^1_6)(a)) = F(g^3_{11})(g^1_6(a)) = F(g^3_{11})(a) = g^1_{11}(a) = a$

ومنه $F(g^3_{11} o g^1_6)(a) = F(g^3_9)(a) = a$

- $F(g^3_{11}) \, o \, F(g^1_6)(b) = F(g^3_{11})(F(g^1_6)(b)) = F(g^3_{11})(g^1_6(b)) = F(g^3_{11})(b) = g^1_{11}(b) = b$

ومنه $F(g^3_{11} o g^1_6)(b) = F(g^3_9)(b) = b$

و بالتالي يجب أن يكون $F(g^3_9) = g^1_9$

9) لأجل $F(g^3_{11}) = g^2_{11}$ $\quad$ $F(g^1_6) = g^1_6$

إن $g^3_{11} o g^1_6 = g^3_9$

- $F(g^3_{11}) \, o \, F(g^1_6)(a) = F(g^3_{11})(F(g^1_6)(a)) = F(g^3_{11})(g^1_6(a)) = F(g^3_{11})(a) = g^2_{11}(a) = b$

ومنه $F(g^3_{11} o g^1_6)(a) = F(g^3_9)(a) = b$

- $F(g^3_{11}) \, o \, F(g^1_6)(b) = F(g^3_{11})(F(g^1_6)(b)) = F(g^3_{11})(g^1_6(b)) = F(g^3_{11})(b) = g^2_{11}(b) = c$

ومنه $F(g^3_{11} o g^1_6)(b) = F(g^3_9)(b) = c$

وبالتالي يجب أن يكون $F(g^3_9) = g^2_9$

10) لأجل $F(g^1_{11}) = g^1_{11}$ $\quad$ $F(g^1_6) = g^2_6$

إن $g^1_{11} o g^1_6 = g^1_9$

- $F(g^1_{11}) \, o \, F(g^1_6)(a) = F(g^1_{11})(F(g^1_6)(a)) = F(g^1_{11})(g^2_6(a)) = F(g^1_{11})(b) = g^1_{11}(b) = b$

ومنه $F(g^1_{11} o g^1_6)(a) = F(g^1_9)(a) = b$

- $F(g^1_{11}) \, o \, F(g^1_6)(b) = F(g^1_{11})(F(g^1_6)(b)) = F(g^1_{11})(g^2_6(b)) = F(g^1_{11})(c) = g^1_{11}(c) = c$

ومنه $F(g^1_{11} o g^1_6)(b) = F(g^1_9)(b) = c$

وبالتالي يجب أن يكون $F(g^1_9) = g^2_9$

11) لأجل $F(g^1_{11}) = g^2_{11}$ $\quad$ $F(g^1_6) = g^2_6$

إن $g^1_{11} o g^1_6 = g^1_9$

- $F(g^1_{11}) \circ F(g^1_6)(a) = F(g^1_{11})(F(g^1_6)(a)) = F(g^1_{11})(g^2_6(a)) = F(g^1_{11})(b) = g^2_{11}(b) = c$

ومنه $F(g^1_{11} o g^1_6)(a) = F(g^1_9)(a) = c$

- $F(g^1_{11}) \circ F(g^1_6)(b) = F(g^1_{11})(F(g^1_6)(b)) = F(g^1_{11})(g^2_6(b)) = F(g^1_{11})(c) = g^2_{11}(c) = a$

ومنه $F(g^1_{11} o g^1_6)(b) = F(g^1_9)(b) = a$

وبالتالي يجب أن يكون $F(g^1_9) = g^3_9$

12) لأجل $F(g^1_{11}) = g^3_{11}$ $\quad$ $F(g^1_6) = g^2_6$

إن $g^2_{11} o g^1_6 = g^2_9$

- $F(g^1_{11}) \circ F(g^1_6)(a) = F(g^1_{11})(F(g^1_6)(a)) = F(g^1_{11})(g^2_6(a)) = F(g^1_{11})(b) = g^3_{11}(b) = a$

ومنه $F(g^1_{11} o g^1_6)(a) = F(g^1_9)(a) = a$

- $F(g^1_{11}) \circ F(g^1_6)(b) = F(g^1_{11})(F(g^1_6)(b)) = F(g^1_{11})(g^2_6(b)) = F(g^1_{11})(c) = g^3_{11}(c) = b$

ومنه $F(g^1_{11} o g^1_6)(b) = F(g^1_9)(b) = b$

وبالتالي يجب أن يكون $F(g^1_9) = g^1_9$

13) لأجل $F(g^2_{11}) = g^2_{11}$ $\quad$ $F(g^1_6) = g^2_6$

إن $g^2_{11} o g^1_6 = g^2_9$

- $F(g^2_{11}) \circ F(g^1_6)(a) = F(g^2_{11})(F(g^1_6)(a)) = F(g^2_{11})(g^2_6(a)) = F(g^2_{11})(b) = g^2_{11}(b) = c$

ومنه $F(g^2_{11} o g^1_6)(a) = F(g^2_9)(a) = c$

$F(g^2_{11}) \circ F(g^1_6)(b) = F(g^2_{11})(F(g^1_6)(b)) = F(g^2_{11})(g^2_6(b)) = F(g^2_{11})(c) = g^2_{11}(c) = b$

ومنه $F(g^2_{11} o g^1_6)(b) = F(g^2_9)(b) = b$

وبالتالي يجب أن يكون $F(g^2_9) = g^3_9$

14) لأجل $F(g^2_{11}) = g^1_{11}$ $\quad$ $F(g^1_6) = g^2_6$

إن $g^2_{11} o g^1_6 = g^2_9$

- $F(g^2_{11}) \circ F(g^1_6)(a) = F(g^2_{11})(F(g^1_6)(a)) = F(g^2_{11})(g^2_6(a)) = F(g^2_{11})(b) = g^1_{11}(b) = b$

ومنه $F(g^2_{11} o g^1_6)(a) = F(g^2_9)(a) = b$

$F(g^2_{11}) \circ F(g^1_6)(b) = F(g^2_{11})(F(g^1_6)(b)) = F(g^2_{11})(g^2_6(b)) = F(g^2_{11})(c) = g^1_{11}(c) = c$

ومنه $F(g^2_{11} o g^1_6)(b) = F(g^2_9)(b) = c$

وبالتالي يجب أن يكون $F(g^2_9) = g^2_9$

15) لأجل $F(g^2_{11}) = g^3_{11}$ $F(g^1_6) = g^2_6$

إن $g^2_{11} o g^1_6 = g^2_9$

- $F(g^2_{11}) \circ F(g^1_6)(a) = F(g^2_{11})(F(g^1_6)(a)) = F(g^2_{11})(g^2_6(a)) = F(g^2_{11})(b) = g^3_{11}(b) = a$

ومنه $F(g^2_{11} o g^1_6)(a) = F(g^2_9)(a) = a$

- $F(g^2_{11}) \circ F(g^1_6)(b) = F(g^2_{11})(F(g^1_6)(b)) = F(g^2_{11})(g^2_6(b)) = F(g^2_{11})(c) = g^3_{11}(c) = b$

ومنه $F(g^2_{11} o g^1_6)(b) = F(g^2_9)(b) = b$

وبالتالي يجب أن يكون $F(g^2_9) = g^1_9$

16) **لأجل $F(g^3_{11}) = g^3_{11}$ $F(g^1_6) = g^2_6$**

إن $g^3_{11} o g^1_6 = g^3_9$

- $F(g^3_{11}) \circ F(g^1_6)(a) = F(g^3_{11})(F(g^1_6)(a)) = F(g^3_{11})(g^2_6(a)) = F(g^3_{11})(b) = g^3_{11}(b) = a$

ومنه $F(g^3_{11} o g^1_6)(a) = F(g^3_9)(a) = a$

$F(g^3_{11}) \circ F(g^1_6)(b) = F(g^3_{11})(F(g^1_6)(b)) = F(g^3_{11})(g^2_6(b)) = F(g^3_{11})(c) = g^3_{11}(c) = b$

ومنه $F(g^3_{11} o g^1_6)(b) = F(g^3_9)(b) = b$

وبالتالي يجب أن يكون $F(g^3_9) = g^1_9$

17) **لأجل $F(g^3_{11}) = g^1_{11}$ $F(g^1_6) = g^2_6$**

إن $g^3_{11} o g^1_6 = g^3_9$

- $F(g^3_{11}) \circ F(g^1_6)(a) = F(g^3_{11})(F(g^1_6)(a)) = F(g^3_{11})(g^2_6(a)) = F(g^3_{11})(b) = g^1_{11}(b) = b$

ومنه $F(g^3_{11} o g^1_6)(a) = F(g^3_9)(a) = b$

- $F(g^3_{11}) \circ F(g^1_6)(b) = F(g^3_{11})(F(g^1_6)(b)) = F(g^3_{11})(g^2_6(b)) = F(g^3_{11})(c) = g^1_{11}(c) = c$

ومنه $F(g^3_{11} o g^1_6)(b) = F(g^3_9)(b) = c$

وبالتالي يجب أن يكون $F(g^3_9) = g^2_9$

18) لأجل $F(g^3_{11}) = g^2_{11}$ $F(g^1_6) = g^2_6$

إن $g^3_{11} o g^1_1 = g^3_4$

- $F(g^3_{11}) \circ F(g^1_6)(a) = F(g^3_{11})(F(g^1_6)(a)) = F(g^3_{11})(g^2_6(a)) = F(g^3_{11})(b) = g^2_{11}(b) = c$

ومنه $F(g^3_{11} o g^1_6)(a) = F(g^3_9)(a) = c$

- $F(g^3_{11}) \circ F(g^1_6)(b) = F(g^3_{11})(F(g^1_6)(b)) = F(g^3_{11})(g^2_6(b)) = F(g^3_{11})(c) = g^2_{11}(c) = a$

ومنه $F(g^3_{11} o g^1_6)(b) = F(g^3_9)(b) = a$

وبالتالي يجب أن يكون $F(g^3_9) = g^3_9$

19) لأجل $F(g^1_{11}) = g^1_{11}$ $F(g^2_6) = g^2_6$

إن $g^1_{11} o g^2_6 = g^2_9$

- $F(g^1_{11}) \circ F(g^2_6)(a) = F(g^1_{11})(F(g^2_6)(a)) = F(g^1_{11})(g^2_6(a)) = F(g^1_{11})(b) = g^1_{11}(b) = b$

ومنه $F(g^1_{11} o g^2_6)(a) = F(g^2_9)(a) = b$

- $F(g^1_{11}) \circ F(g^2_6)(b) = F(g^1_{11})(F(g^2_6)(b)) = F(g^1_{11})(g^2_6(b)) = F(g^1_{11})(c) = g^1_{11}(c) = c$

ومنه $F(g^1_{11} o g^2_6)(b) = F(g^2_9)(b) = c$

وبالتالي يجب أن يكون $F(g^2_9) = g^2_9$

20) لأجل $F(g^1_{11}) = g^2_{11}$ $F(g^2_6) = g^2_6$

إن $g^1_{11} o g^2_6 = g^2_9$

- $F(g^1_{11}) \circ F(g^2_6)(a) = F(g^1_{11})(F(g^2_6)(a)) = F(g^1_{11})(g^2_6(a)) = F(g^1_{11})(b) = g^2_{11}(b) = c$

ومنه $F(g^1_{11} o g^2_6)(a) = F(g^2_9)(a) = c$

- $F(g^1_{11}) \circ F(g^2_6)(b) = F(g^1_{11})(F(g^2_6)(b)) = F(g^1_{11})(g^2_6(b)) = F(g^1_{11})(c) = g^2_{11}(c) = a$

ومنه $F(g^1_{11} o g^2_6)(b) = F(g^2_9)(b) = a$

وبالتالي يجب أن يكون $F(g^2_9) = g^3_9$

21) لأجل $F(g^1_{11}) = g^3_{11}$ $\quad$ $F(g^2_6) = g^2_6$

إن $g^1_{11} o g^2_6 = g^2_9$

- $F(g^1_{11}) \circ F(g^2_6)(a) = F(g^1_{11})(F(g^2_6)(a)) = F(g^1_{11})(g^2_6(a)) = F(g^1_{11})(b) = g^3_{11}(b) = a$

ومنه $F(g^1_{11} o g^2_6)(a) = F(g^2_9)(a) = a$

$F(g^1_{11}) \circ F(g^2_6)(b) = F(g^1_{11})(F(g^2_6)(b)) = F(g^1_{11})(g^2_6(b)) = F(g^1_{11})(c) = g^3_{11}(c) = b$

ومنه $F(g^1_{11} o g^2_6)(b) = F(g^2_9)(b) = b$

وبالتالي يجب أن يكون $F(g^2_9) = g^1_9$

22) لأجل $F(g^2_{11}) = g^2_{11}$ $\quad$ $F(g^2_6) = g^2_6$

إن $g^2_{11} o g^2_6 = g^3_9$

- $F(g^2_{11}) \circ F(g^2_6)(a) = F(g^2_{11})(F(g^2_6)(a)) = F(g^2_{11})(g^2_6(a)) = F(g^2_{11})(b) = g^2_{11}(b) = c$

ومنه $F(g^2_{11} o g^2_6)(a) = F(g^3_9)(a) = c$

- $F(g^2_{11}) \circ F(g^2_6)(b) = F(g^2_{11})(F(g^2_6)(b)) = F(g^2_{11})(g^2_6(b)) = F(g^2_{11})(c) = g^2_{11}(c) = a$

ومنه $F(g^2_{11} o g^2_6)(b) = F(g^3_9)(b) = a$

وبالتالي يجب أن يكون $F(g^3_9) = g^3_9$

23) لأجل $F(g^2_{11}) = g^1_{11}$ $\quad$ $F(g^2_6) = g^2_6$

إن $g^2_{11} o g^2_6 = g^3_9$

- $F(g^2_{11}) \circ F(g^2_6)(a) = F(g^2_{11})(F(g^2_6)(a)) = F(g^2_{11})(g^2_6(a)) = F(g^2_{11})(b) = g^1_{11}(b) = b$

ومنه $F(g^2_{11} o g^2_6)(a) = F(g^3_9)(a) = b$

- $F(g^2_{11}) \circ F(g^2_6)(b) = F(g^2_{11})(F(g^2_6)(b)) = F(g^2_{11})(g^2_6(b)) = F(g^2_{11})(c) = g^1_{11}(c) = c$

ومنه $F(g^{2}_{11}og^{2}_{6})\ (b) = F(\ g^{3}_{9}\ )\ (b) = c$

وبالتالي يجب أن يكون $F(g^{3}_{9}) = g^{2}_{9}$

24) لأجل $F(\ g^{2}_{11}) = g^{3}_{11}$ ، $F(\ g^{2}_{6}) = g^{2}_{6}$

إن $g^{2}_{11}og^{2}_{6} = g^{3}_{9}$

- $F(g^{2}_{11})\ o\ F(g^{2}_{6}\ )\ (a) = F\ (g^{2}_{11})\ (\ F(g^{2}_{6})\ (a)\ ) = F(g^{2}_{11})(\ g^{2}_{6}\ (a)) = F(g^{2}_{11})\ (b) = g^{3}_{11}\ (b) = a$

ومنه $F(g^{2}_{11}og^{2}_{6})\ (a) = F(\ g^{3}_{9}\ )\ (a) = a$

- $F(g^{2}_{11})\ o\ F(g^{2}_{6})\ (b) = F\ (g^{2}_{11})\ (\ F(g^{2}_{6})\ (b)\ ) = F(g^{2}_{11})(\ g^{2}_{6}\ (b)\ ) = F(g^{2}_{11})(c) = g^{3}_{11}(c) = b$

ومنه $F(g^{2}_{11}og^{2}_{6})\ (b) = F(\ g^{3}_{9}\ )\ (b) = b$

ويالتالي يجب أن يكون $F(g^{3}_{9}) = g^{1}_{9}$

25) لأجل $F(\ g^{3}_{11}) = g^{3}_{11}$ ، $F(\ g^{2}_{6}) = g^{2}_{6}$

إن $g^{3}_{11}og^{2}_{6} = g^{1}_{9}$

- $F(g^{3}_{11})\ o\ F(g^{2}_{6}\ )\ (a) = F\ (g^{3}_{11})\ (\ F(g^{2}_{6})\ (a)\ ) = F(g^{3}_{11})(\ g^{2}_{6}\ (a)) = F(g^{3}_{11})\ (b) = g^{3}_{11}\ (b) = a$

ومنه $F(g^{3}_{11}og^{2}_{6})\ (a) = F(\ g^{1}_{9}\ )\ (a) = a$

- $F(g^{3}_{11})\ o\ F(g^{2}_{6})\ (b) = F\ (g^{3}_{11})\ (\ F(g^{2}_{6})\ (b)\ ) = F(g^{3}_{11})(\ g^{2}_{6}\ (b)\ ) = F(g^{3}_{11})(c) = g^{3}_{11}(c) = b$

ومنه $F(g^{3}_{11}og^{2}_{6})\ (b) = F(\ g^{1}_{9}\ )\ (b) = b$

وبالتالي يجب أن يكون $F(g^{1}_{9}) = g^{1}_{9}$

26) لأجل $F(\ g^{3}_{11}) = g^{1}_{11}$ ، $F(\ g^{2}_{6}) = g^{2}_{6}$

إن $g^{3}_{11}og^{2}_{6} = g^{1}_{9}$

- $F(g^{3}_{11})\ o\ F(g^{2}_{6}\ )\ (a) = F\ (g^{3}_{11})\ (\ F(g^{2}_{6})\ (a)\ ) = F(g^{3}_{11})(\ g^{2}_{6}\ (a)) = F(g^{3}_{11})\ (b) = g^{1}_{11}\ (b) = b$

ومنه $F(g^{3}_{11}og^{2}_{6})\ (a) = F(\ g^{1}_{9}\ )\ (a) = b$

- $F(g^{3}_{11})\ o\ F(g^{2}_{6})\ (b) = F\ (g^{3}_{11})\ (\ F(g^{2}_{6})\ (b)\ ) = F(g^{3}_{11})(\ g^{2}_{6}\ (b)\ ) = F(g^{3}_{11})(c) = g^{1}_{11}(c) = c$

ومنه $F(g^{3}_{11}og^{2}_{6})\ (b) = F(\ g^{1}_{9}\ )\ (b) = c$

وبالتالي يجب أن يكون $F(g^{1}_{9}) = g^{2}_{9}$

27) لأجل $F(g^3_{11}) = g^2_{11}$ **$F(g^2_6) = g^2_6$**

إن $g^3_{11} o g^2_6 = g^1_9$

- $F(g^3_{11}) \circ F(g^2_6)(a) = F(g^3_{11})(F(g^2_6)(a)) = F(g^3_{11})(g^2_6(a)) = F(g^3_{11})(b) = g^2_{11}(b) = c$

ومنه $F(g^3_{11} o g^2_6)(a) = F(g^1_9)(a) = c$

- $F(g^3_{11}) \circ F(g^2_6)(b) = F(g^3_{11})(F(g^2_6)(b)) = F(g^3_{11})(g^2_6(b)) = F(g^3_{11})(c) = g^2_{11}(c) = a$

ومنه $F(g^3_{11} o g^2_6)(b) = F(g^1_9)(b) = a$

وبالتالي يجب أن يكون $F(g^1_9) = g^3_9$

28) لأجل $F(g^1_{11}) = g^1_{11}$ **$F(g^2_6) = g^1_6$**

إن $g^1_{11} o g^2_6 = g^2_9$

- $F(g^1_{11}) \circ F(g^2_6)(a) = F(g^1_{11})(F(g^2_6)(a)) = F(g^1_{11})(g^1_6(a)) = F(g^1_{11})(a) = g^1_{11}(a) = a$

ومنه $F(g^1_{11} o g^2_6)(a) = F(g^2_9)(a) = a$

- $F(g^1_{11}) \circ F(g^2_6)(b) = F(g^1_{11})(F(g^2_6)(b)) = F(g^1_{11})(g^1_6(b)) = F(g^1_{11})(b) = g^1_{11}(b) = b$

ومنه $F(g^1_{11} o g^2_6)(b) = F(g^2_9)(b) = b$

وبالتالي يجب أن يكون $F(g^2_9) = g^1_9$

29) لأجل $F(g^1_{11}) = g^2_{11}$ **$F(g^2_6) = g^1_6$**

إن $g^1_{11} o g^2_6 = g^2_9$

- $F(g^1_{11}) \circ F(g^2_6)(a) = F(g^1_{11})(F(g^2_6)(a)) = F(g^1_{11})(g^1_6(a)) = F(g^1_{11})(a) = g^2_{11}(a) = b$

ومنه $F(g^1_{11} o g^2_6)(a) = F(g^2_9)(a) = b$

- $F(g^1_{11}) \circ F(g^2_6)(b) = F(g^1_{11})(F(g^2_6)(b)) = F(g^1_{11})(g^1_6(b)) = F(g^1_{11})(b) = g^2_{11}(b) = c$

ومنه $F(g^1_{11} o g^2_6)(b) = F(g^2_9)(b) = c$

وبالتالي يجب أن يكون $F(g^2_9) = g^2_9$

30) لأجل $F(g^{3}_{11}) = g^{3}_{11}$ **$F(g^{2}_{6}) = g^{1}_{6}$**

إن $g^{1}_{11} o g^{2}_{6} = g^{2}_{9}$

- $F(g^{1}_{11}) \circ F(g^{2}_{6})(a) = F(g^{1}_{11})(F(g^{2}_{6})(a)) = F(g^{1}_{11})(g^{1}_{6}(a)) = F(g^{1}_{11})(a) = g^{3}_{11}(a) = c$

ومنه $F(g^{1}_{11} o g^{2}_{6})(a) = F(g^{2}_{9})(a) = c$

$F(g^{1}_{11}) \circ F(g^{2}_{6})(b) = F(g^{1}_{11})(F(g^{2}_{6})(b)) = F(g^{1}_{11})(g^{1}_{6}(b)) = F(g^{1}_{11})(b) = g^{3}_{11}(b) = a$

ومنه $F(g^{1}_{11} o g^{2}_{6})(b) = F(g^{2}_{9})(b) = a$

وبالتالي يجب أن يكون $F(g^{2}_{9}) = g^{3}_{9}$

31) لأجل $F(g^{2}_{11}) = g^{2}_{11}$ **$F(g^{2}_{6}) = g^{1}_{6}$**

إن $g^{2}_{11} o g^{2}_{6} = g^{3}_{9}$

- $F(g^{2}_{11}) \circ F(g^{2}_{6})(a) = F(g^{2}_{11})(F(g^{2}_{6})(a)) = F(g^{2}_{11})(g^{1}_{6}(a)) = F(g^{2}_{11})(a) = g^{2}_{11}(a) = b$

ومنه $F(g^{2}_{11} o g^{2}_{6})(a) = F(g^{3}_{9})(a) = b$

- $F(g^{2}_{11}) \circ F(g^{2}_{6})(b) = F(g^{2}_{11})(F(g^{2}_{6})(b)) = F(g^{2}_{11})(g^{1}_{6}(b)) = F(g^{2}_{11})(b) = g^{2}_{11}(b) = c$

ومنه $F(g^{2}_{11} o g^{2}_{6})(b) = F(g^{3}_{9})(b) = c$

وبالتالي يجب أن يكون $F(g^{3}_{9}) = g^{2}_{9}$

32) لأجل $F(g^{2}_{11}) = g^{1}_{11}$ **$F(g^{2}_{6}) = g^{1}_{6}$**

إن $g^{2}_{11} o g^{2}_{6} = g^{3}_{9}$

- $F(g^{2}_{11}) \circ F(g^{2}_{6})(a) = F(g^{2}_{11})(F(g^{2}_{6})(a)) = F(g^{2}_{11})(g^{1}_{6}(a)) = F(g^{2}_{11})(a) = g^{1}_{11}(a) = a$

ومنه $F(g^{2}_{11} o g^{2}_{6})(a) = F(g^{3}_{9})(a) = a$

- $F(g^{2}_{11}) \circ F(g^{2}_{6})(b) = F(g^{2}_{11})(F(g^{2}_{6})(b)) = F(g^{2}_{11})(g^{1}_{6}(b)) = F(g^{2}_{11})(b) = g^{1}_{11}(b) = b$

ومنه $F(g^{2}_{11} o g^{2}_{6})(b) = F(g^{3}_{9})(b) = b$

و بالتالي يجب أن يكون $F(g^{3}_{9}) = g^{1}_{9}$

$F(g^2_6) = g^1_6$ 33) لأجل $F(g^2_{11}) = g^3_{11}$

إن $g^2_{11} o g^2_6 = g^3_9$

- $F(g^2_{11})\ o\ F(g^2_6)(a) = F(g^2_{11})(F(g^2_6)(a)) = F(g^2_{11})(g^1_6(a)) = F(g^2_{11})(a) = g^3_{11}(a) = c$
- $F(g^2_{11})\ o\ F(g^2_6)(b) = F(g^2_{11})(F(g^2_6)(b)) = F(g^2_{11})(g^1_6(b)) = F(g^2_{11})(b) = g^3_{11}(b) = a$

ومنه $F(g^2_{11} o g^2_6)(b) = F(g^3_9)(b) = a$

و بالتالي يجب أن يكون $F(g^3_9) = g^3_9$

$F(g^2_6) = g^1_6$ 34) لأجل $F(g^3_{11}) = g^3_{11}$

إن $g^3_{11} o g^2_6 = g^1_9$

- $F(g^3_{11})\ o\ F(g^2_6)(a) = F(g^3_{11})(F(g^2_6)(a)) = F(g^3_{11})(g^1_6(a)) = F(g^3_{11})(a) = g^3_{11}(a) = c$
- ومنه $F(g^3_{11} o g^2_6)(a) = F(g^1_9)(a) = c$
- $F(g^3_{11})\ o\ F(g^2_6)(b) = F(g^3_{11})(F(g^2_6)(b)) = F(g^3_{11})(g^1_6(b)) = F(g^3_{11})(b) = g^3_{11}(b) = a$

ومنه $F(g^3_{11} o g^2_6)(b) = F(g^1_9)(b) = a$

و بالتالي يجب أن يكون $F(g^1_9) = g^3_9$

$F(g^2_6) = g^1_6$ 35) لأجل $F(g^3_{11}) = g^1_{11}$

إن $g^3_{11} o g^2_6 = g^1_9$

- $F(g^3_{11})\ o\ F(g^2_6)(a) = F(g^3_{11})(F(g^2_6)(a)) = F(g^3_{11})(g^1_6(a)) = F(g^3_{11})(a) = g^1_{11}(a) = a$
- ومنه $F(g^3_{11} o g^2_6)(a) = F(g^1_9)(a) = a$
- $F(g^3_{11})\ o\ F(g^2_6)(b) = F(g^3_{11})(F(g^2_6)(b)) = F(g^3_{11})(g^1_6(b)) = F(g^3_{11})(b) = g^1_{11}(b) = b$

ومنه $F(g^3_{11} o g^2_6)(b) = F(g^1_9)(b) = b$

و بالتالي يجب أن يكون $F(g^1_9) = g^1_9$

21) لأجل $F(g^2_6) = g^1_6$ ، $F(g^3_{11}) = g^2_{11}$

إن $g^3_{11} o g^2_6 = g^1_9$

- $F(g^3_{11}) \circ F(g^2_6)(a) = F(g^3_{11})(F(g^2_6)(a)) = F(g^3_{11})(g^1_6(a)) = F(g^3_{11})(a) = g^2_{11}(a) = b$

- ومنه $F(g^3_{11} o g^2_6)(a) = F(g^1_9)(a) = b$

- $F(g^3_{11}) \circ F(g^2_6)(b) = F(g^3_{11})(F(g^2_6)(b)) = F(g^3_{11})(g^1_6(b)) = F(g^3_{11})(b) = g^2_{11}(b) = c$

- ومنه $F(g_{311} o g_{26})(b) = F(g_{19})(b) = c$

- وبالتالي يجب أن يكون $F(g^1_9) = g^2_9$

بما أنه لدينا (43) شعاع مثبت و فق F أي أن F(f)=f و لدينا (11) شعاع غير مثبت وفق F سنحاول إيجاد F(g)

ولهذا سندرس النتائج التي توصلنا إليها من دراسة التراكيب الممكنة

من مثال :

- لأجل $F(g^1_8) = g^1_8$ فإن $F(g^1_{10}) = g^1_{10}$ ومنه $F(g^1_5) = g^1_5$

و لكن لا يوجد أي حالة ل $F(g_2)$ تعطي $F(g^1_5) = g^1_5$

- لأجل $F(g^1_2) = g^1_2$ فإن $F(g^2_5) = g^2_5$ ومنه $F(g^2_{10}) = g^2_{10}$

و لكن لا يوجد أي حالة ل $F(g_8)$ تعطي $F(g^2_{10}) = g^2_{10}$

و بالتالي نجد أنه لا يمكن إيجاد أي Fixob يحقق جميع هذه التراكيب

⟸ لا يوجد Fixob لحالة$_4$ Grph. سوى التشاكل الفئوي الواحدي .

(5-2-4)إيجاد Fixob لحالة$_5$ Grph. :

في حالة الفئة Grph. $_5$ سنذكر بعناصرها

$- G^{1}_{3,3}-\ G^{3}_{3,2}-\ G^{2}_{3,2}-\ G^{1}_{3,2}-\ G^{1}_{3,1}-\ G^{1}_{2,1}-\ G^{1}_{3,0}-\ G^{1}_{2,0}-\ -G_{1}-\ -G_{0}$

$- G^{2}_{4,3}\ \ G^{4}_{4,2}-\ G^{1}_{4,3}\ -\ G^{3}_{4,2}-\ G^{2}_{4,2}-\ G^{1}_{4,2}-\ -\ G^{1}_{4,1}-\ G^{1}_{3,0}-\ G^{1}_{4,0}--\ G^{2}_{3,3}$

$- G^{2}_{4,4}-\ G^{1}_{4,4},\ -G^{10}_{4,3}\ -\ G^{9}_{4,3}-\ G^{8}_{4,3},\ -\ G^{7}_{4,3}\ -\ G^{6}_{4,3},-\ G^{5}_{4,3}-\ G^{4}_{4,3},-\ G^{3}_{4,3}$

$G^{1}_{4,5}\ -G^{11}_{4,4}-\ G^{10}_{4,4},\ -G^{9}_{4,4}\ -\ G^{8}_{4,4}\ G^{7}_{4,4},-\ G^{6}_{4,4}\ -\ G^{5}_{4,4}-\ G^{4}_{4,4},-G^{3}_{4,4}$

$- G^{6}_{4,5}\ -\ G^{5}_{4,5}-\ G^{4}_{4,5}-\ G^{3}_{4,5}-G^{2}_{4,5}$

$.\ G^{4}_{4,6}-\ G^{3}_{4,6}\ -G^{2}_{4,6}-\ G^{1}_{4,6}-\ G^{9}_{4,5}-\ G^{8}_{4,5}-\ G^{7}_{4,5}$

أشعتها (865) شعاعا

ويتضح من خلال المبرهنة التالية أنه لا يوجد سوى التشاكل الفئوي الواحدي كتشاكل مثبت للعناصر(**Fixob**)للفئة$_5$ **Grph.**

(6-2-4) مبرهنة :

لا يوجد Fixob لحالة$_5$. Grph سوى التشاكل الواحدي .

البرهان : لنفرض جدلا أنه يوجد Fixob لحالة$_5$. Grph و أنه ليس التشاكل الواحدي بالتالي فإن مقصوره غلى $_4$. Grph سيكون Fixob (كون أشعة $_4$. Grph موجودة في $_5$. Grph)

و بما أنه لا يوجد أي Fixob لحالة$_4$. Grph سوى التشاكل الواحدي .

وبالتالي نصل إلى تناقض و بالتالي لا يوجد Fixobسوى الواحدي للفئة $_5$. Grph كتشاكل فئوي مثبت للعناصر .

و بالإستقراء الرياضي يتضح أن التشاكل الفئوي الوحيد المثبت للعناصر للفئة $_n$. Grph:n=0,1,2,...... هو التشاكل الفئوي الواحدي

المصطلحات و الرموز

Arc	سهم
Category	الفئة
Digraph	البيان الموجه
Edge	الضلع
Fixob	التشاكل الفئوي المثبت للعناصر
Functor	التشاكل الفئوي
Graph	البيان
Grph	فئة البيانات
Grph$_{\square n}$	الفئات المولدة من البيانات المولدة بالمجموعات المنتهية n
Isomorphism of Graph	التشاكل البياني

in degree	درجة الدخول
multiple arc	سهم مضاعف
multiple edge	ضلع مضاعف
out degree	درجة الخروج
Simpl e graph	البيان البسيط
Supdigraph	البيان الموجه الجزئي
Supgraph	البيان الجزئي
Vertex	العقدة
vertex degree	درجة العقدة

References المراجع

{1}Joan M .Aldous and Robin J.Wilson,Graphs and Applications,An Introductory Approach .

{2}Gros, J .and .Yellen , (1999)"Graph Theory and its Applications" CRCpress,

U.S.A .

{3}Chartrand ,G .and O.R. Oellermann,(1993)."APPlied and Algorithmic Graph Theory " , McGraw – Hill , New York .

{4} Beneck , S, (2004). " Higher Order Domination of Graphs " , Master Thesis,Stellenbosch University , South Africa .

{5}Diestel , R, (2005). " Graph Theory" Springer- Verlag Heidellberg, New York

{6} Hand book of categorical Algebra 1 Basic Category Theory (1994).

{7}pupe and applied mathematics " categories and functors" (1970)

{8} Saunders Mac Lane, Categories for the Working Mathematician, springer-Verlag. (1971)

{9} Kouide , M . and D. p. vestegrd (2006) ." Generalized Connected domination in graphs " , Discrete Mathematics and Theoretical Computer science .

{10} Kenneth H . Rosen " Discrete Mathematics and its Applications. (2003) .